ABSTRACTS

Mass transportation and rough curvature bounds for discrete spaces

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We develop a notion of rough curvature bounds for discrete spaces, based on the concept of optimal mass transportation. These rough curvature bounds will depend on a real parameter $h \geq 0$, which should be considered as a natural length scale of the underlying discrete space or as the scale on which we have to look at the space. Mass transportation and convexity properties of the relative entropy will be studied along h-geodesics. Instead of midpoints of a given pair of points x_0, x_1 we look at h-midpoints which are points y with $\mathsf{d}(x_0, y) \leq \frac{1}{2}\,\mathsf{d}(x_0, x_1) + h$ and $\mathsf{d}(x_1, y) \leq \frac{1}{2}\,\mathsf{d}(x_0, x_1) + \frac{h}{2}$.

We prove that an arbitrary metric measure space (M, d, m) has curvature $\geq K$ (in the sense of K.-T. Sturm) provided it can be approximated by a sequence (M_h, d_h, m_h) of ('discrete') metric measure spaces with h-curvature $\geq K_h$ with $K_h \to K$ as $h \to 0$. The curvature bounds will also be preserved under the converse procedure: Given any metric space (M, d, m) with curvature $\geq K$ and any $h > 0$ we define approximating standard discretizations (M_h, d, m_h) of (M, d, m) with h-curvature $\geq K$.

Finally, we apply our results to concrete examples of planar graphs.

Decomposition of finely superharmonic functions

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The talk is based on a joint work with **Stephen J. Gardiner**.

Question 1. Suppose that u is a non-negative finely superharmonic function on a fine domain, and that the only non-negative finely harmonic minorant of u is 0. Does it follow that u is a fine potential?

This has been a central open question in fine potential theory since its development at the beginning of the 1970's. As a test case towards understanding Question 1, Fuglede posed (in 1987) another problem that is of interest in its own right, concerning minimal harmonic functions associated with irregular boundary points of (Euclidean) domains.

Question 2. Let x_0 be an irregular boundary point of a domain Ω, and let u be a positive harmonic function on Ω. Then u has a fine limit, l say, at x_0. If u is minimal and $l = +\infty$, does it follow that u is a multiple of the fine Green function for $\Omega \cup \{x_0\}$ with pole at x_0?

It follows from work of Brelot that the answer to Question 2 is "yes" in the case of plane domains Ω. The lecture will present answers to both questions in higher dimensions.

Convexity of limits of harmonic measures

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The talk is based on a joint work with **Ivan Netuka**.

It is shown that, given a point $x \in \mathbb{R}^d$, $d \geq 2$, and open sets $U_1, \ldots, U_k$ containing x, any convex combination of the harmonic measures for x with respect to U_n, $1 \leq n \leq k$, is the limit of a sequence of harmonic measures for x with respect to open subsets W_m of $U_1 \cup \cdots \cup U_k$ containing x. This answers a question raised in connection with Jensen measures.

More generally, we prove that, for arbitrary measures on an open set W, the set of extremal representing measures, with respect to the cone of continuous potentials on W or with respect to the cone of continuous functions on the closure of W which are superharmonic W, is dense in the compact convex set of all representing measures.

This is achieved approximating balayage on open sets by balayage on unions of balls which are pairwise disjoint and very small with respect to their mutual distances and then shrinking these balls in a suitable manner.

The results are presented simultaneously for the classical case and for the theory of Riesz potentials.

Hot spots conjecture and maximum principles

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The Hot Spots Conjecture was proposed by Jeffrey Rauch at a conference at Tulane University, in 1974.

Suppose that D is an open connected bounded subset of $\mathbb{R}^d$, $d \geq 1$. Let $u(t,x)$, $t \geq 0$, $x \in D$, be the solution of the heat equation $\frac{1}{2}\Delta u = \frac{\partial u}{\partial t}$ in D with the Neumann boundary conditions and the initial condition $u(0,x) = u_0(x)$, that is, $u(t,x)$ is a solution to the following initial-boundary value problem:

$$\begin{cases} \frac{1}{2}\Delta_x u(t,x) = \frac{\partial u}{\partial t}, & (t,x) \in (0,\infty) \times D \\ \frac{\partial u}{\partial n}(t,x) = 0, & t > 0, x \in \partial D \\ u(0,x) = u_0(x), & x \in D. \end{cases}$$

The physical interpretation of Hot Spots conjecture is that for most initial conditions (i.e. functions $u_0(x)$), if m_t denotes the place where the function $u(t,x)$ attains its maximum (for $t > 0$ fixed, as a function of $x \in \bar{D}$), then $m_t \to \partial D$ as $t \to \infty$, that is, the "hot spots" migrate towards the boundary of the domain, and a similar statement holds for the "cold spots".

The analytic counterpart of the conjecture is that if φ_2 denotes the second Neumann eigenfunction for the domain D, that is if φ_2 satisfies

$$\begin{cases} \frac{1}{2}\Delta\varphi_2 + \lambda_2\varphi_2 = 0 & \text{in } D \\ \frac{\partial \varphi_2}{\partial n} = 0 & \text{on } \partial D \end{cases},$$

(λ_2 is the second Neumann eigenvalue for D, that is the smallest non-zero eigenvalue of the Laplaceian in D for which a non-trivial solution to the above boundary problem exists), then φ_2 satisfies a strong maximum principle: it attains its maximum and a minimum only on the boundary of the domain D, unless it is identically constant in D.

This is a natural generalization of the maximum principle for harmonic functions, if we take into consideration that the first Neumann eigenfunction $\lambda_1 = 0$ and a first Neumann eigenfunction is a harmonic function:

$$\begin{cases} \frac{1}{2}\Delta\varphi_1 + \lambda_1\varphi_1 = 0 & \text{in } D \\ \frac{\partial\varphi_1}{\partial n} = 0 & \text{on } \partial D \end{cases},$$

We will present a (partial) resolution of the Hot Spots Conjecture, for convex domains D having one axis of symmetry. The arguments involved are from Stochastic processes (Brownian motion, stochastic calculus) and Complex analysis.

References

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Some remarks on a nonlinear semigroup acting on positive measures

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Taking as starting point a recent paper, [M. Baake: Recombination Semigroups on Measure Spaces, *Monatsh.Math.* **146** (2005), 267–278], where a semigroup of positive, nonlinear operators acting on measures is studied, we present a few related results of potential theory.

First, the associated resolvent is computed. In a second part, the invariant and excessive elements with respect to the resolvent/semigroup, are determined. Finally, the problem of extending these results to a more general setting, through the product, is formulated.

Singular solutions of the perturbed logistic equation in anisotropic media

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We consider a linear perturbation of the logistic equation in anisotropic media and we establish a necessary and sufficient condition for the existence of a positive solution blowing-up on the boundary of the domain. The proofs relies on adequate comparison techniques for nonlinear elliptic equations. The main result of this talk answers to a question raised by Prof. H. Brezis.