A Sequent Approach to Logical Foundations of Automated Reasoning

ABSTRACT

The course is devoted to logical foundations to the constructions of computer-oriented calculi (methods) for logical inference search in 1st-order classical logic and some non-classical logics. The course is based on original investigations of its author, which are related to the sequent approach to the construction of logics. Attention is mainly concentrated on ideas leading to the construction of certain inference search methods disregarding sophistical technical details. Main results are included in the course. The methods explicitly are described for classical logic (without equality) and are provided with some comments both for using equality and for the development of mechanized intuitionistic inference search.

As a scientific discipline, logic is usually understood as one that studies general laws of reasoning. Furthermore, when one has something to do with reasoning, he usually adverts to (analogues of) concepts of a sentence and of its modal- (i.e. truth-) value.

Therefore, we can concern logic with two concepts: truth (of assertion under consideration) and/or its provability (in a certain logical system reflecting ways of reasoning about a world under examination). These concepts have been investigated intensively for centuries, by philosophers, linguists, and mathematicians. The purpose of this lectures is by no means to give a general account of such studies. Instead, its purpose is to focus on a mathematically well-defined logical system known as first-order classical logic in its sequent treatment and to prove some basic properties of this system permitting to construct various computer-oriented methods of logical inference systems (classical and non-classical).

The sequent approach proposed for logics in the framework of the course has a number of advances in comparison with others, among which are:

1. It can be considered as the development of Gentzen's approach in the direction of increasing the efficiency of usual sequent calculi by means of using special deductive technique, mainly, the original notion of substitution admissibility introduced by the author.

2. It gives a possibility to formulate sequent forms (without skolemization) of Herbrand's theorem for first-order both classical and intutionistic logics. All well known forms of Herbrand's theorem including the forms A, B, and C from Herbrand's original paper can be obtained as special cases of the proposed ones.

3. These forms serve as a base for the construction of different computer-oriented sequent-type calculi, which can be "rewritten" in the form of tableau calculi for both classical and intuitionistic logics.

4. Along with the sequent and tableau calculi, the approach gives a possibility, in the case of classical logic, to make the unified description of such wide-used methods of inference search as the resolution method, the inverse method, and their various modifications.