THE SEMINAR "QUALITATIVE THEORY OF DIFFERENTIAL EQUATIONS AND CONTROL THEORY" – 60 YEARS

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The scientific seminar with the initial subject *Qualitative theory of differential equations* , organized jointly by the *Institute of Mathematics of the Romanian Academy* and the *Department of Differential and Integral Calculus of the School (Faculty) of Mathematics and Physics of the Bucharest University* at the initiative and under the leadership of **Professor Aristide Halanay** (1924-1997) started in February 1953 and takes place weekly and continuously since then. Some years later, seminars devoted to differential equations began at the Univ. of Cluj-Napoca under the leadership of Prof. D.V. Ionescu (in 1955) and at the Univ. of Iasi under the leadership of Prof. C. Corduneanu (in 1957). Their activity continues in various forms up to now.

Organized following the model of the homonyme seminar lead by **V. V. Nemytskii** (whose Ph. D. student Halanay had been between 1948-1952) at the *School(Faculty) of Mathematics and Mechanics of the Moscow University*, the seminar was dedicated to those problems of differential equations considered as belonging to the qualitative theory : boundedness, oscillations, (almost)-periodicity and stability. *At that time these problems were most actual and most studied of the field and further evolution of the topics of the seminar confirmed the fact that its half-century orientation was permanently*

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targetted to the most actual and interesting problems and methods of the area of differential equations and their applications in science, engineering and economics.

Together with Professor Halanay we may list among the first participants of the seminar F. Albrecht(1926-1992), I. Barbălat(1907-1988), I. Berstein, T. Gane, M. Reghiş, Şt. Sandor, D. Wexler(1933-1991). The main interest during the first period of existence of the seminar was for *periodic and almost periodic solutions in a topological framework* : we may cite here the applications of Ważewski method by Halanay and Barbălat, fixed point applications, generalization by T. Gane of a result due to Marachkoff a.o. Some of the results obtained during this early period are included in the book of Halanay *Introduction to qualitative theory of differential equations* published in Romanian in 1956.

Openness to the applied problems suggested by engineering brought to the attention of seminar's members the very actual (in the '50ies and '60ies) *problem of the absolute stability of control systems.* Exactly at that time the seminar is joined by V. M. Popov, young electrical engineer (in 1956) and T. Morozan (in 1959). A survey of Halanay (published in 1959 but written most probably in 1957) mentions some already obtained new results *via* the direct method of Liapunov by V. M. Popov, M. Reghiş and D. Wexler.

It was in this seminar where the *famous frequency domain inequality for the absolute stability* of V. M. Popov was presented by the author himself in 1958. This completely new approach in absolute stability generated further a long line of research which is still active, under various names (hyperstability, dissipativity/passivity) in Romania and worldwide. Within the seminar this line of research was represented by I. Barbălat (the author of a worldwide known *asymptotics lemma*, currently applied in most studies of absolute stability and passivity), T. Morozan (with a result *remarked and cited* by S. Lefschetz; his research will then continue on *stochastic differential equations*), Halanay himself (on absolute stability and hyperstability of *systems with deviated arguments* and of *discrete time systems*), S. Gheţaru (electrical engineer who joined the seminar in 1966 and worked on hyperstability of *discrete time and anticipative (non-causal) systems*) and Vl. Răsvan (electrical engineer who joined the seminar in 1967 and worked on *stability and stabilization of discrete time systems and systems with deviated argument*).

Understanding of Popov's methods lead in a natural way to the topics of *controlled systems* which had become extremely popular at the beginning

of the '60ies. Several aspects were considered. Among them, and widely cultivated in the '60ies and '70ies, was the *optimal control of dynamical systems*. Here we can mention the research of Halanay (*necessary conditions, differential games, relaxed control, linear quadratic optimization problem for discrete-time and time delay systems*), C. Vârsan (who joined the seminar in 1962 , on *necessary and sufficient conditions of optimality*; later he turned to *stochastic control*), St. Mirică (who joined the seminar in 1965 and directed his research to *optimal synthesis and geometric theory*; later he continued on *basic theory for discontinuous differential equations occurring in various applications including optimal control*), C. Drăguşin (who joined the seminar in 1965 and studied the *Pareto optimum*) and Maria Giurgiu (who joined the seminar in 1966 and worked on *linear -quadratic optimal control for partial differential equations*).

Another topic of control theory, stimulated in a way also by the theory of V. M. Popov, was the *structural theory of linear systems*, considered by V. Prepeliţă (who joined the seminar in 1965) and Vlad Ionescu (1938-2000) (electrical engineer who joined the seminar in 1977; he wrote several monographs on this subject).

Starting with 1975 the interest for *engineering applications* increased among the members of the seminar (partly due to social pressure but partly due to the strong belief of Professor Halanay that engineering is a valuable source of models and problems for the field of differential equations). The most remakable effect of this aspect was the research directed to *singular perturbations* and *discretization as connected to computer control*. These directions were covered by V. Drăgan (who joined the seminar in 1975) and M. Popescu (hydraulic engineer who joined the seminar in 1976) together with Professor Halanay. Due especially to M. Altăr, the analysis of *economic dynamics* was introduced in the seminar from the late '60ies of the previous century. This start was followed by a certain cooperation led by Professor Halanay, between the members of the seminar and interested researchers coming from the fields of economic mathematical modeling. It is a rather interesting aspect to mention the continuity of this field of Mathematical Economics over several decades, as a reflex of the rapid changes of the economic vision worldwide. Based on his "stochastic background", C. Vârsan developed during the last two decades a research programme which is partly incorporated in the Ph.D. theses realized under his guidance.

The 3d topic in control that established itself at the beginning of the

'80ies was the so-called H_∞-*control* with all its auxiliaries. The field was tackled by Halanay (who used the time domain and time varying framework in order to put all research and existing results on a sound mathematical background) and Vlad Ionescu, who rebuilt the entire theory around the positivity theory of V. M. Popov, generalized to the case of indefinite sign. Among the co-workers of V. Ionescu, the young electrical engineers M. Weiss (who joined the seminar in 1989), C. Oară (who joined the seminar in 1991) and R. Ştefan (also with the seminar since 1992) dealt with *specific aspects of the generalized* H_∞ *theory* (Pritchard-Salamon systems, spectral factorization, scattering theory) including *numerical issues*, while Radu Bălan (who joined the seminar in 1992) was concerned with *nonlinear control*, later turning to *wavelet and frame analysis*. A somehow peculiar direction of the nonlinear control was for a while the so called *geometric control*: several presentations dealt with this topic, made by V. Ionescu, Th. Hangan, V. Brânzănescu and P. Flondor. A. Stoica (electrical engineer who joined the seminar in 1992) did research within the same framework and studied *robust control systems* and their applications.

A permanent line of research in the seminar has been the *extension and development of the approaches and of the results from ordinary differential equations to other classes of dynamical systems*. With respect to this, the *analysis in the stochastic framework of the most topics* discussed in the seminar was a main research line for T. Morozan, C. Vârsan, Al. Ghiţă (electrical engineer who joined the seminar in 1969 and was concened with *reliability optimization* from stochastic viewpoint), V. Drăgan and A. Stoica. We may add here the research on *discrete-time systems* (i.e. described by difference equations) covered by Professor Halanay, D. Wexler, T. Morozan (within stochastic framework), V. Drăgan, A. Stoica, Vl. Răsvan. *Systems with deviated argument* (both time delay and neutral) were considered by Professor Halanay and Vl. Răsvan and the dynamic and controlled systems described by partial differential equations by Maria Giurgiu, D. Tiba (who joined the seminar in 1977) and C. Marinov (electrical engineer who joined the seminar in 1979).

The "historical", evergreen topics of Liapunov stability, self sustained (mainly based on bifurcation theory) and forced oscillations as applied to various classes of systems (including those generated by engineering and non-engineering fields, biology and medecine) are as such a constant direction in the seminar, covered by T. Morozan, Vl. Răsvan and Andrei Halanay.

The seminar as a scientific institution has been *opened to young researchers* , being helpful for their training and their career. The founders of the seminar were rather young at that time. Most of the participants joined the seminar in their graduation year or as Ph. D. students and *presented their theses in front of the seminar.* Besides this there were presented in the seminar *the theses belonging to the Ph. D. students of the participants of the seminar* and other theses. We may mention here a list (far of being complete) of the theses presented in the seminar: *M. Altăr, Angela Beju, A. Bătătorescu, Ş. Bolintineanu, Mona Doroftei, B. Iftimie, D. Lambadarie, M. Olteanu, Gh. Potcovaru, Judita Samuel, T. Vasilache.* We have to add here the theses presented in the '70ies by Chilean and Vietnamese Ph. D. students of Professor Halanay : *M. Barrahona, H. Burgos, J. Serrano, L. Vergara; N. Cang, P. T. Nhu and L. Doâng.* We also add to this list (already mentioned that it is far from being complete) the most recent Ph.D. theses presentations under the scientific guidance of C. Vârsan: *Marinela Marinescu, Daniela Ijacu, M. Nica* and the activity of *Diana Merlusca* who is currently preparing her Ph.D. thesis under the supervision of D. Tiba.

The seminar has been and is a *prestigious scientific institution both at the national and international levels.* Among the speakers with an established scientific reputation we may mention (again without aiming to completeness of the list) *C. Corduneanu*, *V. Barbu* and *Theodor Hangan* and from abroad *L. Neustadt* and *G. Sell* (from USA); *I. Gohberg, M. A. Krasnosel'skii and K. S. Sibirskii* (from USSR); *A.J.Pritchard* (from Great Britain); *Cz. Olech* (from Poland); *P. Brunovsky, J. Kurzweil, St. Schwabik, M. Tvrdý, O. Vejvoda, I. Vrkoć* (from Czechoslovakia) a.o.

The scientific results of the research performed under the auspices of the seminar have been published in hundreds of papers that appeared in the most valuable mathematical journals and in dozens of books; these books cover all the topics of the seminar (but not only !) and have been published by the most prestigious publishing houses in Romania and throughout the world; these books are read and cited worldwide.