Abstract:

The Stone representation theorem is one of the recognized landmarks of the modern mathematics. It states that an algebrization of propositional logic is dual to Stone topological spaces. Jonsson and Tarski extended this to modal logic and certain topological spaces with additional operators. Plotkin and Smyth emphasized the duality between state-transformer semantics and predicate-transformer semantics, Kozen discovered such a duality for probabilistic transition systems. Recently several authors have emphasized the duality between logics and transition systems from a coalgebraic perspective. Mislove et al. have found a duality between labelled Markov processes and C*-algebras based on the closely related classical Gelfand duality. These dualities are important because they allow us to transport theorems between mathematical fields apparently independent.

We take the challenge of identifying a Stone-type duality for Markov Processes. We define Aumann algebras, an algebraic analog of probabilistic modal logic. An Aumann algebra consists of a Boolean algebra with operators modeling probabilistic transitions. We prove a Stone-type duality theorem between countable Aumann algebras and countably-generated continuous-space Markov processes. Our results subsume existing results on completeness of probabilistic modal logics for Markov processes.

This talk resumes a joint work of mine with Dexter Kozen (Cornell Univ.), Kim Larsen (Aalborg Univ.) and Prakash Panangaden (Mc Gill Univ.), which was presented to the Twenty-Eight ACM/IEEE Symposium on Logics in Computer Science, LICS2013.