Report of Activity, Eugen Mihailescu

September 2013

We present briefly some of my research accomplishments, citations, and invitations for research visits. In the last 30 years, one of the most successfull developments in Dynamical Systems has been the use of thermodynamical formalism for problems in fractal dimension and smooth ergodic theory. For example several of the recent Fields medalists, S. Smirnov, E. Lindenstrauss, C. McMullen, J.C Yoccoz, work in these and related fields.

Most of my research was in smooth ergodic theory and thermodynamic formalism, applied to dimension theory in hyperbolic dynamics, fractal geometry, conformal and complex dynamics, and stochastic properties for various types of measure-preserving systems.

In 1999 I finished my PH.D at University of Michigan, whose mathematics department ranks consistently among the 10 best in the USA. My PH.D supervisor was Prof. John Erik Fornaess, one of the top mathematicians in the world, and also one of the founders of higher dimensional complex dynamics.

After the PH.D, I applied and obtained a visiting assistant professorship at Texas A&M University, one of the top 20 mathematics departments in USA; I was there in the period 1999-2001. Later, in 2002 I had a visiting professorship at Univ. of North Texas, where I worked with Prof. M. Urbanski, one of the best known researchers in the world in Dynamical Systems.

I also held other academic positions for various durations at several prestigious universities and institutes abroad, among them: Institut des Hautes Études Sciéntifiques (IHÉS), Paris, November-Dec 2013 and Aug-Sept 2014; Univ. of Bremen, Germany, March-May 2012; Max Planck Institut, Bonn, May 2011; Univ. of North Texas, February-March 2007; Instituto de Matematica Pura e Aplicada (IMPA), Rio de Janeiro, Brazil, February 2008; Erwin Schrödinger Math Institute, Vienna, Austria, June 2008; Univ. of North Texas, Sep. 2004; MSRI, Berkeley, Sep. 2004; Univ. of Pisa-Centro de Giorgi, Pisa, Italy, Nov. 2004; Inst Mathematics Polish Academy (IMPAN), Warsaw, Apr. 2004; Scuola Normale Superiore, Pisa, Italy, Apr. 2002, etc.

I have published so far over 34 research papers, most of them in known ISI journals, such as: Math. Annalen (2013, 1999), International Math Res Notices (2013), Ergodic Theory Dynam Syst (2011, 2011, 2002), Math Zeitschrift (2011), Discrete and Cont Dynam Syst (2012, 2008, 2006, 2001), Bulletin London Math Soc (2010), J Stat Physics (2013, 2011, 2010), Commun Contemp Math (2004), Mathematical Proceed Cambridge (2010), Monatshefte Math (2012), Canadian J Math (2008), Proceed American Math Soc (2013, 2011), Nonlinear Analysis (2010), Houston J Math (2005), J Math Analysis and Appl (2012), etc.

Together with M. Urbanski (U. North Texas), we published in 2013 a section of the book on "Fractal Geometry and Dynamics in Pure and Applied Mathematics", in the Contemporary Mathematics series, vol. 600, from the American Mathematical Society. I also had several other invited articles, in Discrete Cont. Dynam. Systems, Oberwolfach Reports, Révue Roumaine Math. Pures Appl., and in Ann. Univ. Bucharest.

In 2013, I wrote also my Habilitation Thesis, entitled "Smooth Ergodic Theory and Dimension in Dynamical Systems", at IMAR, Bucharest.

I collaborated over time with well-known mathematicians such as: M. Urbanski (U. North Texas), J.E Fornaess (U. Michigan), B. Stratmann (U. Bremen), M. Abate (U. Pisa), M. Denker (U. Göttingen), M. Roychowdhury (U. Texas-Edinburg), Z. Nitecki (Tufts U.), etc.

I was on the Global Organizing Committe of the 8-th International Conference on Dynamical Systems and Differential Equations, Dresden, Germany, 2010, a conference with over 1000 participants. In 2004-2005, I co-organized and participated in the "Dynamics and Ergodic Theory" Exchange Program between the Polish and the Romanian Academies.

I was grant director (PI) for a PN II-Idei PCE research project, from CNCSIS (Romania) during 2009-2011, leading a team of 4 members. I was principal investigator, or member also in other research projects at IMAR. In 2007, I held a research fellowship within an NSF grant at Univ. North Texas (director M. Urbanski). CNCSIS awarded me several prizes, for papers published in the period 2008-2012.

Since 2011, I am on the Editorial Board of the ISI journal Discrete and Continuous Dynamical Systems-series S (USA).

Our results have been cited, for example: by Okuyama in Michigan Math J. 2010; by Sumi and Urbanski in Advances in Math, 2013; by S. Nakane in Ergodic Th Dynam Syst, 2013; by Diller et al., in Annales Sciéntifiques de l'École Normale Supérieure, 2010; by Dinh and Sibony in Lecture Notes in Math 2010; by Chen et al., in Pacific J Math, 2012; by Wang et al. in Nonlinear Analysis, 2013; by Gkana et el. in International J Nonlinear Sci, 2013; by Nitecki in Real Analysis Exchange 2003/2004, etc. Citations are detailed at the end of this Report.

One of my research directions has been the theory of *hyperbolicity for endomorphisms*. For non-invertible maps, the dynamical behaviour is very different than in the case of diffeomorphisms and there appear many new phenomena, for which one has to create new methods. For endomorphisms, local unstable manifolds do not form a foliation, there may be uncountably many local unstable manifolds through a point, and they may intersect both inside and outside the fractal Λ.

We gave new examples of endomorphisms with strong non-invertible behaviour, and studied the consequences of hyperbolicity in their case. A family of such examples, which present uncountably many unstable manifolds through certain points, and for which the *fiber Hausdorff dimension* also behaves differently than in the diffeomorphism case, was given in our paper in Math. Zeit 2011. In that paper we found a *new type of Newhouse phenomenon* for intersections of Cantor sets in fibers, which translates into a class of skew products which are far from being homeomorphism on the respective basic sets, but also far from being constant-to-1. Our class of examples is significant since it gives endomorphisms on actual fractals (non-Anosov), and involves a completely different technique to obtain examples with *uncountably many unstable manifolds* through a given point, than the toral examples given by Przytycki. We studied also holomorphic endomorphisms on $\mathbb{P}^2\mathbb{C}$, and complex disks given by stable/unstable manifolds.

We also studied thermodynamical formalism and statistical properties for invariant measures on *folded fractals* (for eg in our paper from Discrete Cont Dyn Syst 2006). In this theory, we employed frequently *equilibrium (Gibbs) measures* of Hölder continuous potentials, which were shown to have good properties on iterates of Bowen balls. The Sinai-Ruelle-Bowen (or SRB) measures were introduced in the ground-breaking works of Sinai, Ruelle and Bowen and, in the case of hyperbolic attractors Λ they represent the physically observable measures that give the asymptotic distribution of forward trajectories of Lebesgue-almost all points in a neighbourhood of Λ. We introduced in our paper from J. Stat Phys 2010, the new notion of *inverse SRB measures* for non-invertible dynamics, and studied it in the framework of equilibrium measures. Inverse SRB measures are not simply SRB measures for inverse function, since in this case the functions are not invertible. Instead they give *asymptotic distributions* of inverse branches of iterates, and satisfy a *modified Pesin entropy formula*.

Jointly with Fornaess in our paper from Math Annalen 2013, we found results about the *pointwise dimensions of equilibrium measures* in the case of holomorphic endomorphisms on complex projective spaces. We proved that the measure of maximal entropy *for the restriction* to a terminal set, is in fact a *wedge product* of two positive closed currents. This shows that the measure of maximal entropy on a terminal saddle set (which is singular w.r.t the Green measure), has a geometric description with closed currents.

We found also results about *Jacobians of equilibrium measures*, and about mixing of any order and decay of correlations for equilibrium measures. In a joint paper with M. Urbanski, J. Stat Phys 2013, we solved the problem of *negative entropy production* for measure-preserving endomorphisms on Lebesgue spaces, which is related to the work of Ruelle.

Another direction was about applications of thermodynamic formalism in *dimension theory for non-invertible maps*. We showed that the problem of dimension (Hausdorff, box, stable, etc.) is *very different* than in the diffeomorphism case, and provided also perturbation examples with strange dynamical behaviour, obtained by me and Urbanski in our paper from Houston J Math, 2005. We also gave applications of a new notion of *inverse pressure* introduced by Mihailescu and Urbanski in our paper from Commun Contemp Math, 2004. We applied this notion to various estimates for global and sectional dimensions, in a paper from Canadian J Math 2008, and then in another paper from Math Proceed Cambdridge, 2010.

Together with M. Urbanski, and B. Stratmann, we solved the difficult dimension problems (see related problems of Verjovsky-Wu, Manning-McCluskey) for the *stable intersections*, in relation to the preimage counting function, in our papers from Bull London Math Soc 2010, and International Math Res Notices 2013. We employed careful combinatorial study, together with new methods of smooth ergodic theory and fractal decomposition, in order to prove that if stable dimension takes its minimal, resp. maximal value as zero of the pressure, then the number of preimages remaining in Λ is maximal, resp. minimal. The paper from BLMS is also related to some questions of Simon. The IMRN paper is related (but in a different setting), to questions of Schief, and to a problem of self-similar sets with overlaps of Falconer.

In our paper from Ergodic Th Dynam Syst 2002, we solved a problem of Fornaess and Sibony, proving that for s-hyperbolic holomorphic endomorphisms on $\mathbb{P}^2$, the *interior of the set* K^- (similar to the set of points with bounded inverse iterates from the Hénon automorphisms case), is empty. Then, in our paper from Math Proc Cambridge Phil Soc, 2010, we obtained a detailed description of the structure of the global unstable set $W^u(\hat{\Lambda})$ of a saddle basic set Λ, giving thus also more information about the Hausdorff and sectional dimensions of K^-.

In our paper from Math Annalen 1999, we studied also dynamical *actions of groups of holomorphic automorphisms* on Kobayashi hyperbolic Stein manifolds Ω, and showed that the number of periodic points of all periods, is finite in certain cases. Then we studied S^1-actions generated by f on Ω and showed that, under certain *non-resonance conditions* on the eigenvalues of the derivative of f at 0, there exists exactly one fixed point of f in Ω. In the same paper from Math Annalen we also generalized this uniqueness result, to another case, when a *cohomology groups condition* was satisfied. Our results answer thus some questions related to rotation domains. Concrete examples were given as well in that paper.

We studied applications of a notion of *transversality* for parametrized families of hyperbolic skew products, in a paper joint with Urbanski, Discrete Cont Dynam Syst, 2008. Our result is related to some questions and work of Peres, Simon, Solomyak, etc.

We also found several unexpected relations between ergodic theory of invariant measures and dimension theory on folded fractals. In our paper from Monatsh Math, 2012, we proved results about the *coding and mixing properties* for measure-preserving endomorphism. This problem is subtle, and as Parry and Walters showed, it is profoundly different than in the case of automorphisms. For example 2-sided Benoulli shifts can be classified by entropy alone (Ornstein), however this is not true for 1-sided shifts, for which there is no such general classification.

In our paper from Monatsh Math, 2012, we solved a problem of Dajani and Hawkins in the setting of hyperbolic endomorphisms which are non-expanding on basic sets, showing the *non-existence of generating Rokhlin partitions*.

We found also a surprising *geometric flattening phenomenon* in our paper from J Stat Phys 2011, that proves, in short, that if stable dimension at a point of the fractal is zero, then the whole fractal is contained in a global unstable manifold (if it is connected). Moreover answering some questions of Parry, Bruin, Hawkins, etc., we showed (also in the paper from Monatsh Math 2012), that there exists a connection between *1-sided Bernoullicity* and the *expanding property*.

Then, in our paper from Ergodic Th Dynam Syst 2011, we proved that the *stable conditional measures* of certain equilibrium measures are geometric measures, and that these conditional measures maximize, in a Variational Principle for the stable dimension. We also proved that the exponent of these geometric measures is the stable dimension.

We also studied dimension theory for *iterated function systems* (IFS). In general, IFS were studied under separation conditions, such as Open Set Condition (Hutchinson, Falconer, Moran, Mauldin, etc). However if Open Set Condition does not hold, then the dimension theory is much harder and there are few results in the literature. In a paper from Proc Amer Math Soc, 2011, Mihailescu and Urbanski attacked this problem using thermodynamic formalism and a preimage counting function, together with an analysis of the equilibrium measures on *limit sets of finite IFS with overlaps.* We proved that, if the dimension of the limit set is minimal possible as a zero of the corresponding pressure function, then the system is as far from Open Set Condition (OSC) as possible. This analysis was later extended by us, in Contemp Math 2013, to the case of IFS with *countably many generators and arbitrary overlaps*, where even more new phenomena appear, as compared to the case of infinite IFS with OSC studied by Mauldin-Urbanski; for eg. the limit set is no longer compact, there appear conditions at the boundary at infinity, etc.

In our paper from J. Math Analysis and Appl. 2012, we applied techniques of thermodynamic formalism and ergodic theory to problems from *economic dynamics* such as overlapping generations models, a cobweb model with adaptive adjustments, heterogeneous markets, etc.

Jointly with Roychowdhury, we found in 2013, a construction of a family of non-stationary infinitely generated Moran fractals, determined by *asymptotic frequencies.* For these fractals, we built a form of thermodynamic formalism. We applied this construction in order to relate the dimension theory for these non-stationary Moran fractals, to the ergodic properties of f-expansions, digits behaviour, etc. In particular these relations are found for digits in m-expansions, β-expansions, Bolyai-Rényi expansions, and continued fraction expansions.

Citations.

At the moment there exist over 27 citations (without self-citations), most of which are in well-known ISI journals, and some in non-ISI journals. Below are most of citations.

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