ACTIVITY STATEMENT - Delia Ionescu-Kruse

• I graduated from the University of Bucharest, Faculty of Mathematics in 1994. In 1995 I completed a Master's degree in Geometry and in 2002 I received a PhD degree in Mathematics at the University of Bucharest. The title of my PhD dissertation was "Applications of the geometry in Logunov's relativistic theory of gravitation", the thesis advisors were Prof. Dr. S. Ianuş, from the University of Bucharest and Senior Researcher E. Soós from the Institute of Mathematics of the Romanian Academy (IMAR). All my degrees were obtained with excellent grades. In 2005 I obtained the Qualification at the Maître de Conférences positions (Section 25) in France.

• Between 1995 and September 2005 I held Assistant and Lecturer positions at the Department of Mathematics, Technical University of Civil Engineering Bucharest (UTCB). This was also a good period for further study, training and working on my research projects. I have benefited from different fellowships (3 months in 1997 at the University of Perpignan, France; 1 month in 1998 at the Summer School on Differential Geometry at Cortona, Italy; 1 month in 2005 at the Summer School and Conference on Poisson Geometry, Abdus Salam ICTP-Trieste, Italy). In the period March 2002 - August 2004 I held a MASIE (Mechanics and Symmetry and Europe) postdoctoral position at the Zentrum Mathematik der Technischen Universität München (TUM), Germany.
Since October 2005, I have been Senior Researcher III at IMAR.

RESEARCH ACTIVITY

• I have done research on various topics. The research area that I have focused on while PhD student was *relativistic theories of gravitation.* During the postdoctoral position, I was working in *geometric mechanics, Birkhoffian systems*, *nonholonomic geometry and mechanics.* My current research interests are *partial differential equations in fluid mechanics*, *water waves*, *geometric methods in hydrodynamics.*

• On the basis of my PhD thesis I have published a chapter in a volume edited by Lazăr Dragoş, Editura Academiei Române. I have published or in press 21 research articles, 19 of them being in ISI journals with impact factor higher than 0.5 (see the list of publications). In conference proceedings with peer review I have published 4 papers. See also "Fişa de verificare a îndeplinirii standardelor minimale", for the indexes I= 17.293 and I_{recent}=13.555.
My papers have been cited in journal articles and books. There are 66 citations without self citations (see the list of citations). See also "Fişa de verificare a îndeplinirii standardelor minimale" for the index C=63.

• I communicated my results at international scientific meetings (see the list of invited and contributed talks in the CV).
I was invited at prestigious universities and research centers: University of Vienna, Erwin Schrödinger Institute (ESI) Vienna, École Polytechnique Fédérale de Lausanne, Lund University, Trinity College Dublin, Université d'Aix-Marseille I, Institute of High Energy Physics (IHEP), Protvino, Moscow Region (see the CV).

• In 2008 I received the *Dimitrie Pompeiu* prize of the Romanian Academy.

▶ *PDEs in Fluid Mechanics*, *Water Waves*, *Geometric Methods in Hydrodynamics.*

• The classical water-wave problem concerns the two-dimensional irrotational flow of a perfect, incompressible fluid under the influence of gravity. The fluid motion is described by the Euler equations in a domain bounded below by a rigid horizontal bottom and above by a free surface. The full Euler equations are often too complicated to analyze directly, their complexity led mathematicians and physicists to derive simpler sets of equations likely to describe the dynamics of the water-waves equations in some specific physical regimes. Small-amplitude, long-wavelength (or shallow water) waves are approximated by weakly nonlinear long waves models such as the Korteweg-de Vries (KdV) and Boussinesq equations. These equations have led researchers to understand better the physical behaviour of nonlinear waves and to develop new mathematical theories to explain their behaviour. However, the weakly nonlinear models are not valid for large amplitude waves for which nonlinear effects become more important. Relaxing the assumption that waves have small amplitude leads to the higher-order nonlinear long wave models such as Green-Naghdi (GN) equations (1976, J. Fluid. Mech.).

The KdV equation is the most famous and extensively studied equation in the class of completely integrable nonlinear partial differential equations arising in shallow water theory which captures the existence of solitary waves and possesses solitons - localized solutions which interact strongly with one another but retain their form after an interaction. Another development of models for water waves was initiated in order to gain insight into wave breaking, one of the most fundamental aspects of water waves. In contrast to KdV, two recently derived nonlinear integrable equations, the Camassa-Holm (CH) equation (1993, Phys. Rev. Letters) and the Degasperis-Procesi (DP) equation (1999, "Symmetry and Perturbation Theory", World Scientific) model breaking waves, that is, smooth solutions that develop singularities in finite time, the solution being bounded but its slope becoming unbounded. For alternative derivations of the CH equation within the shallow water regime see also Johnson (2002, J. Fluid Mech.), Dullin, Gottwald and Holm (2003, Fluid Dyn. Res.), **Ionescu-Kruse (2007, J. Nonlinear Math. Phys.)**, Constantin and Lannes (2009, Arch. Ration. Mech. Anal.). In this last article it is also proved that the DP equation arises in the modeling of the propagation of the shallow water waves over a flat bed. There is a large literature devoted to the remarkable features of the CH and DP equations.

Arnold initiates in the 60-70s the use of geometric methods in describing the equations of an incompressible ideal fluid in a bounded domain without free boundary. This description of incompressible ideal fluids consists in formulating the facts on an infinite dimensional configuration space transferring results from the finite dimensional case of Lie group theory and of classical Riemannian geometry. Thus, the Euler equations are geodesic equations for the right-invariant metric on the group of volume-preserving diffeomorphisms. This study was rigorously carried out by Ebin and Marsden (1970, Ann. of Math.). To appreciate the geodesic formulation it is useful to recall that the geodesic spray is a second-order equation, and the partial differential equation becomes an ordinary evolution equation in time on the tangent bundle of the configuration manifold. In this case, one can appeal to the ordinary differential equations literature for existence and other properties of an initial value problem. The geometric point of view is also useful in the study of stability issues. The CH equation describes a geodesic flow for the right-invariant metric on the one dimensional central extension of the group of diffeomorphisms of the circle (see Misiolek (1998, J. Geom. Phys.) and Constantin and

Kolev (2003, Comment. Math. Helv.)). The variational formulation gives a meaning to the geodesic equation on the group of diffeomorphisms. For an irrotational unidirectional shallow water flow, **Ionescu-Kruse (2007, J. Nonlinear Math. Phys.)** derived the CH equation by a variational method in the Lagrangian formalism.

A key quantity in fluid dynamics is the curl of the vector field, called vorticity. Vorticity is adequate for the specification of a flow: a flow which is uniform with depth is described by a zero vorticity (irrotational case), a constant non-zero vorticity corresponds to a linear flow and a non-constant vorticity indicates highly sheared flows. The CH equation modelling water waves moving over a shear flow was considered by Johnson (2003, Fluid Dynam. Res.) by an asymptotic expansion procedure and **Ionescu-Kruse (2007, Discrete Contin. Dyn. Syst.)** by a variational method in the Lagrangian formalism. For a non-constant vorticity unidirectional shallow water flow, **Ionescu-Kruse (2007, Discrete Contin. Dyn. Syst.)** showed that the displacement of the free surface from the flat state satisfies a generalized CH type equation.

The GN equations model shallow-water waves whose amplitude is not necessarily small and represent a higher-order correction to the classical shallow-water system. They have nice structural properties that facilitate the derivation of the simplified model equations in the shallow water regime: KdV, CH and DP equations arise as approximations to the GN equations cf. the discussion in Constantin and Lannes (2009, Arch. Ration. Mech. Anal.). The variational derivation of the GN equations is due to **Ionescu-Kruse (2012, J. Nonlinear Math. Phys.)**. The second equation of the GN system is a transport equation, the free surface is advected, or Lie transported (in the geometry language), by the fluid flow. **Ionescu-Kruse (2012, J. Nonlinear Math. Phys.)** showed that the first equation of the GN system yields the critical points of an action functional in the space of paths with fixed endpoints, within the Lagrangian formalism. Firstly, within the Eulerian formalism, it was considered the Lagrangian function integrated over time in the action functional to have the traditional form, that is, the kinetic energy minus the potential energy. Then, this Lagrangian from the Eulerian picture was transported to the tangent bundle which represents the velocity phase space in the Lagrangian formalism, the transport being made taking into account the second equation of the GN system. It is important to point out that in both formalisms the Lagrangians obtained are not metrics.

In the case that the parameter κ from the CH equation is equal to zero, this equation has peakon solutions: solitons with a sharp peak, so with a discontinuity at the peak in the wave slope. A two-component generalization of this peakon CH equation was derived by Olver and Rosenau (1996, Phys. Rev. E) by using geometric bi-Hamiltonian methods. Alternative geometric derivations of this system, are provided by Liu and Zhang (2005, J. Geom. Phys.), Chen, Liu and Zhang (2006, Lett. Math. Phys.) (in this paper the system is called the 2CH system) and Falqui (2006, , J. Phys. A: Math. Gen.). One of the equations of this two-component system is the peakon CH equation minus a term which only depends on the second variable. Let us denote this system by 2CH(-) system. So far, there is no physical interpretation of the 2CH(-) system. For the choice of the plus sign in front of the term which only depends on the second variable, the system, which we denote now by 2CH(+), may be regarded as a model of shallow water waves (see Constantin and Ivanov (2008, Phys. Lett. A)). In Constantin and Ivanov (2008) this system is derived from the GN equations using expansions of the variable in terms of physical parameters. **Ionescu-Kruse (2013, Appl. Anal.)** derived for propagation of irrotational shallow water waves over a flat bed the nonlinear integrable 2CH(+) system by a variational approach in the Lagrangian formalism. The Lagrangian used in the variational derivation is not a metric. It was also underlined that the 2CH(-) system is obtained if instead of the Lagrangian one considers in the action functional the total energy at the free surface.

In the very recent paper **Ionescu-Kruse (2013, Quart. Appl. Math)**, a new two-component system modelling shallow water waves is derived by the use of a variational approach in the Lagrangian framework. In addition, the Hamiltonian structure of the system is elucidated and the explicit solitary waves are found. The type of considerations made in this paper proved also very useful (in similar contexts) to qualitative studies of some model equations. For example, in the derivation of criteria for global existence and blow-up of solutions as well as in studies of the propagation speed for some model equations for shallow water waves, see e.g. the papers Constantin (2000, Ann. Inst. Fourier), Constantin and Escher (2000, Math. Z.), Gui and Y. Liu (2011, Quart. Appl. Math.), Constantin (2005, J. Math. Phys.), Henry (2005, J. Math. Anal. Appl.).

• Notice that there are only a few explicit solutions to the full nonlinear water-wave problems. For periodic gravity water waves in water of infinite depth, Gerstner constructed an explicit solution in 1802. This solution was independently re-discovered later by Rankine (1863, Phil. Trans. R. Soc. A). Modern detailed descriptions of this wave are given in the recent papers Constantin (2001, J. Phys. A) and Henry (2008, J. Nonlinear Math. Phys.). Gerstner's wave is a two-dimensional wave given in the Lagrangian description, by following the evolution of individual water particles. The motion of the water body induced by the passage of Gerstner's wave is rotational, it occurs in a flow with a specific non-constant vorticity. The fact that this flow is very special is confirmed also by the fact that this is the only steady flow satisfying the constraint of constant pressure along the streamlines, cf. Kalisch (2004, J. Nonlinear Math. Phys.). Beneath Gerstner's wave it is possible to have a motion of the fluid where all particles describe circles with a depth-dependent radius. The fact that for Gerstner's waves the fluid particles move on circles is in agreement with the classical description of the particle paths within the framework of linear water-wave theory. In the framework of linear water-wave theory, after the linearization of the governing equations for water waves, the ordinary differential equation system which describes the path of a particle turns out to be again nonlinear. In the first approximation of this nonlinear system, one obtained that all water particles trace closed, circular or elliptic, orbits (see any classical book on water waves). While in this first approximation all particle paths appear to be closed, Constantin and Villari (2008, J. Math. Fluid Mech.) showed, using phase-plane considerations for the nonlinear system describing the particle motion, that in linear periodic gravity water waves no particles trajectory is actually closed, unless the free surface is flat. Each particle trajectory involves over a period a backward/forward movement, and the path is an elliptical arc with a forward drift; on the flat bed the particle path degenerates to a backward/forward motion. Beside the phase-plane analysis, the exact solutions of the nonlinear system describing the particle motion, allow a better understanding of the dynamics. **Ionescu-Kruse (2008, J. Nonlinear Math. Phys.)** obtained the exact solutions of the nonlinear differential equation system which describes the particle motion in small-amplitude irrotational shallow water waves and showed that there does not exist a single pattern for all particles: depending on the strength of the underling uniform current, some particle trajectories are undulating curves to the right, or to the left, others are loops with forward drift, and others are not physically acceptable, in the last case it seems necessary to study the full

nonlinear problem.
The same type of results hold for the governing equations without linearization. Analyzing a free boundary problem for harmonic functions in a planar domain, Constantin (2006, Invent. Math.), Constantin and Strauss (2010, Comm. Pure Appl. Math.) showed that there are no closed orbits for symmetric periodic steady gravity waves (Stokes waves) travelling over a flat bed. While in periodic waves within a period each particle experiences a backward-forward motion with a forward drift, Constantin and Escher (2007, Bull. Amer. Math. Soc.) showed that in a solitary water wave there is no backward motion: all particles move in the direction of wave propagation at a positive speed, the direction being upwards or downwards if the particle precedes, respectively, does not precede the wave crest.

There are also studies of particle paths for rotational waves. Within the linear theory, by using phase-plane considerations for the nonlinear system describing the particle motion, Ehrnström and Villari (2008, J. Differential Eqs.) found that for positive constant vorticity, the behavior of the streamlines is the same as for the irrotational waves, though the physical particle paths behave differently if the size of the vorticity is large enough. For negative vorticity they showed that in a frame moving with the wave, the fluid contains a cat's-eye vortex (see Majda and Bertozzi, 2002, Cambridge Texts Appl. Math.). The paper by Wahlen (2009, J. Differential Eqs.) which contains an existence result for small-amplitude solutions, based on local bifurcation theory, showed also that the predictions for negative vorticity from Ehrnström and Villari (2008, J. Differential Eqs.) in the linear theory are true. An alternative approach to the existence result in Wahlen (2009, J. Differential Eqs.) for small-amplitude steady waves with constant vorticity was very recently proposed by Constantin and Varvaruca (2011, Arch. Ration. Mech. Anal.). For small-amplitude shallow-water waves with vorticity and background flow **Ionescu-Kruse (2009, Nonlinear Anal.-Theor.)** found the exact solutions and showed that depending on the relation between the initial data and the constant vorticity some particles trajectories are undulating curves to the right, or to the left, others are loops with forward drift, or with backward drift, others can follow peculiar shapes. Removing the shallow-water restriction, **Ionescu-Kruse (2012, Commun. Pure Appl. Anal.)** provided explicit solutions for the nonlinear system describing the motion of the particles beneath small-amplitude gravity waves which propagate on the surface of a constant vorticity flow. It is proved that all the paths are not closed curves. Some solutions can be expressed in terms of Jacobi elliptic functions, others in terms of hyperelliptic functions. New kinds of particle paths are obtained (see also **Ionescu-Kruse (2010, J. Nonlinear Math. Phys.)**). In **Ionescu-Kruse (2012, Commun. Pure Appl. Anal.)** two linearizations used in the study of small-amplitude long waves on a constant vorticity flow: one made around still water, the other one made around a laminar flow, are also compared.

The fluid dynamics at the propagation of small-amplitude gravity waves over irrotational deep water was first investigated in Constantin, Ehrnström and Villari (2008, Nonlinear Anal. Real World Appl.). By a phase-plane analysis it is shown that no particle trajectory is actually closed. Within the full nonlinear framework similar conclusions hold for symmetric periodic steady gravity waves travelling over water of infinite depth (see Henry (2006, Int. Math. Res. Not.)). In order to obtain quite precise information about the shape of the particle paths below small-amplitude gravity waves travelling on irrotational deep water, in **Ionescu-Kruse (2013, J. Math. Fluid Mech.)** analytic solutions of the nonlinear differential equation system describing the particle motion are

provided. It is shown that none of these solutions is a closed curve. Some particle trajectories are peakon-like, others can be expressed with the aid of the Jacobi elliptic functions or with the aid of the hyperelliptic functions. Some solutions have vertical asymptotes in the positive direction. The particle seems to be shot out from the flow, this feature could reflect the wave-breaking phenomenon.
In the papers **Ionescu-Kruse (2012, Commun. Pure Appl. Anal.)** and **Ionescu-Kruse (2013, J. Math. Fluid Mech.)** the stagnation points for the considered problems are also investigated. The stagnation points are points where the vertical component of the fluid velocity field is zero while the horizontal component equals the speed of the wave profile. They are of special interest because they are points where the flow characteristics often change.

Below small-amplitude capillary-gravity water waves (the influence of gravity and the effects of surface tension are taken into account), Henry (2007, Phil. Trans. R. Soc. A), by using phase-plane considerations, and **Ionescu-Kruse (2009, Wave Motion)**, **Ionescu-Kruse (2010, Nonlinear Anal. Real World Appl.)**, by founding exact solutions of the nonlinear differential equation system which describes the particle motion, obtained that the particle trajectories are not closed. In the two latest papers the required computations involve elliptic integrals of the first kind, the Legendre normal form and a solvable Abel differential equation of the second kind.

Very recently, **Ionescu-Kruse** and Matioc address the issue of particle paths in equatorial water waves with constant vorticity (for the two-dimensional geophysical water-wave problem, see, for example, the books by Pedlosky, "Geophysical fluid dynamics" (1979) and by I. Gallagher and L. Saint-Raymond, "On the influence of the Earth's rotation on geophysical flows" (2007)) and for the equatorial f-plane approximation see also the paper Constantin (2012, Geophys. Res. Lett.). Under the assumption of small amplitude, the surprising outcome is that the geophysical effects are barely noticeable in the velocity field but appear in the pressure and in the dispersion relation. Their paper *Small-amplitude equatorial water waves with constant vorticity: dispersion relations and particle trajectories*, supported by the ERC Advanced Grant "Nonlinear studies of water flows with vorticity" (NWFV), has been recently accepted for publication in **Discrete Contin. Dyn. Syst. A**.

► *Geometric Mechanics, Birkhoffian Systems, Nonholonomic Geometry and Mechanics.*

• The Birkhoffian formalism is an alternative approach, to the Lagrangian and Hamiltonian ones, in the study of a wide class of dynamical systems, among them the nonholonomic systems, the degenerate systems and the dissipative ones. This is a global formalism for the dynamics of implicit systems of second order differential equations on a manifold, developed by Kobayashi and Oliva (2003, Resenhas IME-USP) following Birkhoff's ideas presented locally in his classical book "Dynamical Systems" from 1927. In order to formulate these ideas in a coordinate free fashion, one considers the formalism of 2-jets. The space of configurations is a smooth m-dimensional differentiable connected manifold and the covariant character of the Birkhoff generalized forces is obtained by defining the notion of elementary work, called Birkhoffian, a special Pfaffian form defined on the 2-jet manifold. The dynamical system associated to this Pfaffian form is a subset of the 2-jet manifold which defines an implicit second order ordinary differential system. Beside the intrinsic (coordinate free) concepts of reversibility, reciprocity, regularity, conservation of energy introduced in Kobayashi and Oliva (2003, Resenhas IME-USP), **Ionescu** and

Scheurle **(2007, Z. Angew. Math. Phys.)** introduced the notion of time-dependent Birkhoffian and they derived a general balance law for an associated energy function, and **Ionescu (2006, J. Geom. Phys.)** introduced the notion of dissipative Birkhoffian.

There were also others ideas of extending the Lagrangian and the Hamiltonian frameworks to include dissipation, degeneracies, and the nonholonomic constraints. In the 90s a generalization of the Hamiltonian framework has been developed in a series of papers. This generalization, which is based on the geometric notion of generalized Dirac structure (see Courant (1990, Trans. Amer. Math. Soc.) and Dorfman (1987, Phys. Lett. A), gives rise to implicit Hamiltonian systems (see, for example, the papers by Maschke and van der Schaft (1995, Archiv für Elektronik und Übertragungstechnik), van der Schaft (1998, Rep. Math. Phys.). Applications to nonholonomic systems and electrical circuits (see Bloch and Crouch (1999, Differential Geometry and Control, Proceedings of Symposia in Pure Mathematics, in: Amer. Math. Soc.), Maschke and van der Schaft (1995, Archiv für Elektronik und Übertragungstechnik)) illustrate this theory. The notion of implicit Lagrangian system has been developed by Yoshimura and Marsden (2006, J. Geom. Phys.) Nonholonomic mechanical systems and degenerate Lagrangian systems such as LC circuits can be systematically formulated in the implicit Lagrangian context in which Dirac structures are also used.

The Birkhoffian formalism in the context of electrical circuits was discussed by **Ionescu** and Scheurle **(2007, Z. Angew. Math. Phys)** for the case of LC circuits, and **Ionescu (2006, J. Geom. Phys.)** for the case of RLC circuits. An LC/RLC circuit, with no assumptions placed on its topology, will be described by a family of Birkhoffian systems, parameterized by a finite number of real constants which correspond to initial values of certain state variables of the circuit. It is shown that the Birkhoffian system associated to an LC circuit is conservative. Under certain assumptions on the voltage-current characteristic for resistors, it is shown that a Birkhoffian system associated to an RLC circuit is dissipative. For LC/RLC networks which contain a number of loops formed only from capacitors, the Birkhoffian associated is never regular. A procedure to reduce the original configuration space to a lower dimensional one, thereby regularizing the Birkhoffian, is presented as well. In order to illustrate the results, specific examples are discussed in detail.

For RLC electrical networks, Brayton and Moser (1964, Quart. Appl. Math.) proved under a special hypothesis, that there exists a mixed potential function which can be used to put the system of differential equations describing the dynamics of such a network, into a special form. The hypothesis they made is that the currents through the inductors and the voltages across the capacitors determine all currents and voltages in the circuit via Kirchhoff's law. The mixed potential function is constructed explicitly only for the networks whose graph possesses a tree containing all the capacitor branches and none of the inductive branches, that is, the network does not contain any loops of capacitors or cutsets of inductors, each resistor tree branch corresponds to a current-controlled resistor, each resistor co-tree branch corresponds to a voltage-controlled resistor . Making different assumptions on the type of admissible nonlinearities in the circuit, this mixed potential function is used to construct Liapunov-type functions to prove stability. Smale (1972, J. Diff. Geom.) also develops the differential equations for nonlinear RLC electrical circuits and illustrates these equations through a series of examples. He builds on the work of Brayton and Moser but he is able to treat more general equations. A large part of the paper illustrates these equations by means of examples and discusses stability properties of the examples. **Ionescu-Kruse (2007, J. Geom. Phys.)** considers the concepts and the direct theorems of stability in the sense of Liapunov within the framework of Birkhoffian dynamical systems on manifolds and applies the theory to electrical circuits. For linear and nonlinear LC and RLC electrical networks, Liapunov-type functions are constructed in order to prove stability or asymptotic stability under certain conditions.

• From the mathematical point of view nonholonomic constraints pose challenging questions. There are different approaches, from the geometric point of view or from the control theory point of view, to these nonintegrable constraints and a large literature on this subject. Very important results on nonholonomic geometry and its connections with mechanics are obtained in the first half of the twentieth century and they are due to Vrânceanu, Synge, Schouten and Wagner. The approach by Vrânceanu (see, for example, "Les espaces non holonomes", Memorial des Sciences Mathematiques, Fascicule LXXVI, Paris, 1936) is based on Cartan's method of moving frame. In the book "Geometry of Riemannian spaces. Lie Groups: History, frontiers and applications, XIII , 1983", pag. 217, Cartan says: "Up to now we have used, almost exclusively, the natural frames attached at each point in a given system of coordinates in space. But it might be more convenient to use locally Cartesian frames that are more appropriate to the nature of the problems outlined, and not necessarily with respect to the coordinates chosen. Each of these frames is defined by its origin and n linear independent basis vectors". Vrânceanu showed that if in a Riemannian space one gives a system of Pfaff equations not completely integrable, that is a nonholonomic space, then "il est possible d'introduire un parallélisme dans le sens de Levi-Civita, de manière que, *à* chaque système non holonome *à* liaisons indépendantes du temps, on peut attacher un espace non holonome, dont les géodésique (courbes auto-parallèles), sont aussi les trajectoires sans forces du système mécanique considéré". Using the concept of the nonholonomic space introduced by Vrânceanu, **Ionescu** and Soós **2001, Annals of the University of Timişoara, Mathematics and Computer Science series**) gave a geometric interpretation of simultaneity of infinitesimal close events in a non-inertial frame in the special relativity theory.

Another approach, based on Ehresmann connection, is that by Bloch, Krishnaprasad, Marsden, Murray (1996, Arch. Rational Mech. Anal.). Ehresmann connection is used to model the constraints and the curvature of this connection enters into Lagrange's equations. Unlike the case of standard constraints, the presence of symmetries in the nonholonomic case may or may not lead to conservation laws. When the nonholonomic connection is a principal connexion for the given symmetry group, Bloch et al. showed how to perform a Lagrangian reduction in the presence of nonholonomic constraints. In order to avoid some assumptions in Bloch et al., in 2002 I started together with Prof. Dr. T. Raţiu (EPFL) to study the nonholonomic systems in the presence of symmetries in Vrânceanu's framework. Our goal was to compare and unify these two, apparently quite different, approaches. At that time, there were still several problems to be resolved in order to be able to give a common formulation. We have so far succeeded in identifying several common geometric structures underlying these two approaches. We have also worked on different examples, among them was Chaplygin's skate. Unfortunately, till now there is no paper published on this subject.

► *Relativistic Theories of Gravitation.*

The relativistic theory of gravitation (RTG) was constructed by Logunov and his coworkers in the 80s as a field theory of the gravitational field in the framework of special relativity (see, for example, the book by A. A. Logunov "Relativistic Theory of Gravity and Mach Principle", Horizons in World Physics, volume 215, Nova Science Publishers, Inc., Commack, New York, 1998). The Minkowskian space-time is a fundamental space that incorporates all physical fields including gravitation. The gravitational field is described by a second-order symmetric tensor whose action generates an effective pseudo-Riemannian space-time. The behavior of matter in the Minkowskian space-time under the influence of the gravitational field is equivalent with its behavior in the effective pseudo-Riemannian space-time. In order to obtain the physically meaningful gravitational fields, the principle of causality (that is, the events separated by spatial intervals in the effective pseudo-Riemannian space-time are also separated by spatial intervals in the Minkowski space-time) must be satisfied.

In the framework of RTG, **Ionescu** and Soós, **(2000, Rev. Roumaine Math. Pures Appl.)** and **Ionescu (2003, Theor. Math. Phys.)** studied the gravitational field generated by a charged mass point having mass m and electric charge q. In order to find in RTG the pseudo-Riemannian metric they solved the non-linear system of differential equations formed by RTG's equations and Maxwell's equations, plus the constraint equations yielded by the causality principle. The problem of finding this field in Einstein's general relativity was solved by Nordström and Jeffrey (see, for example the book by Wang, "Mathematical Principles of Mechanics and Electromagnetism, Part B: Electromagnetism and Gravitation", 1979). **Ionescu (2003, Int. J. Nonlinear Mech.)** made a comparative analysis in Einstein's general relativity and RTG, of the motion of another charged mass point in the gravitational field produced by the charged mass point m, q (an "electrogravitational" Kepler problem). For the so-called elliptic motion, the first and second order approximate solutions are found in the two theories and the results are compared with each other. In an approximation of the solution to second order the advance of perihelion differs in the two theories.

Ionescu and Soós **(2000, Proceedings of the XXIII International Workshop on High Energy Physics and Field Theory, Protvino (Russia)** and **Ionescu (2002, Theor. Math. Phys.)** presented the importance of the causality principle in RTG, in finding the physically meaningful gravitational fields. Some classical problems like Bogorodskii's homogeneous gravitational field, that is, the field generated by a system of mass homogenously distributed on a plane (1971, "Gravity", Section 17, Naukova Dumka, Kiev), Taub's empty universe (1951, Ann. Math.) and Gödel' s rotating universe (1950, Proc. Intern. Congr. Math. Cambridge, Mass.) are considered in RTG's framework. The obtained results differ from those given in Einsteins relativity theory.

Bucharest, 14.09.2013 Signature,