Costin VÎLCU --- Activity Report

In the first, main part of this report I'll briefly present my research results, while in the last part I'll give a list of attended international conferences. The references below are either to the list of my papers, or to the small list at the end of the first part.

I. RESEARCH RESULTS

After two small papers in differential geometry [A18], [BC3], I started to study convex surfaces.

Farthest points on convex surfaces. Consider a convex surface S and the mapping F associating to each point x in S the set of farthest points (i.e., at maximal intrinsic distance) from x. I disproved a conjecture of H. Steinhaus, stating that *the sphere is characterized by F single-valued and involutive* [A17]. In [A15] we provide a large class of surfaces on which F is single-valued and involutive, and show that there are point-symmetric surfaces of revolution with F single-valued but not involutive. In [A8] we provide a sharp, sufficient condition for a point y on a convex surface S to be a farthest point on S; sufficient conditions are derived, to guarantee that a convex cap contains at least one farthest point. In [A11] we provide two large classes of convex hypersurfaces with the mapping F single-valued and involutive; we also show that a convex body is smooth and has constant width if its double has F as above; additional conditions are given, to characterize the balls. Other papers in similar directions are [A4], [A5], [A7], [A14], [A16].

The study of farthest points is essentially related to distance functions and geodesic segments.

Quasigeodesics on convex surfaces. In [A13] we obtain inequalities relating the length of simple closed (quasi)geodesics on a convex surface S, to the area of S and its diameter. In [A5] we obtain a comparison type result, involving geodesic quadrilaterals, and apply it to locate farthest points on convex surfaces.

Farthest points (=global maxima) lead to local maxima for intrinsic distance functions and, more generally, to critical points for distance functions.

Another extension of the framework was the passage from convex surfaces to Alexandrov surfaces with curvature bounded below, as defined for example by [Burago-Gromov-Perelman 92].

Recall that, by Alexandrov's existence theorem and Pogorelov's rigidity theorem, any Alexandrov surface with non-negative curvature bound and homeomorphic to the sphere can be embedded in $\mathbf{R}^3$ as a convex surface, unique up to an isometry of the ambient space.

Critical points for distance functions on Alexandrov surfaces. In [A2] we show that, *for any compact Alexandrov surface S and any point y in S, there exists a point x in S for which y is a critical point; moreover, uniqueness characterizes the surfaces homeomorphic to the sphere among smooth orientable surfaces*. In [A12], I find properties of the sets M_y^{-1} of all points on a compact orientable Alexandrov surface S, the distance functions of which have a common maximum at y in S; for example, the number of components in M_y^{-1} is at most $\max\{1,10g-5\}$, with g the genus of S.

The mapping F above can be similarly defined on compact Alexandrov surfaces; it is for some surfaces single-valued and a homeomorphism, while for other surfaces it is not single-valued, and not surjective. It is still unknown whether, say in the convex case, the second class is dense. In [A12] we show that, for a C^2-metric on both surfaces and the space of surfaces, the first class has nonempty interior; we describe various properties of the sets of critical points, and of relative and absolute maxima of distance functions, and find several connections between them; a particular Tannery surface belonging to the boundary of both classes is presented.

Among the simplest examples of Alexandrov surfaces, one finds doubles of convex bodies. See for example [Shiohama-Tanaka 96].

Properties of typical degenerate convex surfaces. In [A9], I initiated the study of typical (in the sense of Baire category) degenerate convex surfaces; i.e., surfaces obtained by gluing together two isometric copies of typical convex bodies; various properties are given, concerning geodesics on, and distance functions from points in, typical degenerate convex surfaces.

It is known that the space of convex surfaces is a Baire space, and several properties of *most* convex surfaces have been obtained, see for example [Gruber 93].

Baire categories for Alexandrov surfaces. In a recent paper [P2], we establish that the space of Alexandrov surfaces is a Baire space, and consider conical points and curvatures on most Alexandrov surfaces. In an almost completed manuscript, with J. Rouyer, we show that the existence of simple closed geodesics on most Alexandrov surfaces depends on the curvature bound and the topology of the considered surfaces.

Polyhedral convex surfaces are, at the same time, a particular class of convex surfaces and a main object of study for people working in *Computational Geometry*.

Polyhedral convex surfaces. For almost twenty years, only two general methods were known to unfold the surface *P* of any convex polyhedron to a simple polygon in the plane: the source unfolding and the star unfolding, both with respect to a point *x* in *P*. We extended these methods to the star unfolding with respect to *quasigeodesic loops* [A6], and to the source unfolding with respect to closed, polygonal, convex curves [P6]. Very recently, yet another method was discovered [Demaine-Lubiw 12]. A further study of the curves employed in [P6] lead to [A1].

In [BC1] we give a first, partial answer to an open question by [Demaine-O'Rourke 07].

A basic tool for studying distance functions is the cut locus; introduced by H. Poincaré in 1905, it gained since then an important place in Global Riemannian Geometry.

Cut locus structures on graphs. The *cut locus* the point *x* in the surface *S* is the set of all extremities (different from x) of maximal (with respect to inclusion) geodesic segments starting at *x*. In a series of papers [P3-5], [A3], we show that *every connected graph can be realized as the cut locus of some point on some complete, compact and connected Riemannian surface*. We also study the stability and the generic behavior of such realizations. Starting from the above result, we introduce a new structure on graphs, called cut locus (CL-) structure. We find upper bounds on the number of (orientable) CL-structures on a given graph.

References:

[Burago-Gromov-Perelman 92] Y. Burago, M. Gromov and G. Perelman, *A. D. Alexandrov spaces with curvature bounded below*, Russian Math. Surveys 47 (1992), 1-58.

[Demaine-Lubiw 12] E. D. Demaine, A. Lubiw: *A Generalization of the Source Unfolding of Convex Polyhedra*. LNCS 7579, 185-199, Springer Verlag, 2012.

[Demaine-O'Rourke 07] Erik D. Demaine and Joseph O'Rourke, *Geometric Folding Algorithms: Linkages, Origami, Polyhedra*, Cambridge University Press (2007).

[Gruber 93] P. Gruber: *Baire categories in convexity*. In: Gruber, P.,Wills, J. (eds.) *Handbook of Convex Geometry* vol. B, 1327–1346. North-Holland (1993).

[Shiohama-Tanaka 96] K. Shiohama, M. Tanaka: *Cut loci and distance spheres on Alexandrov surfaces*. Actes de la Table Ronde de Géométrie Différentielle (Luminy, 1992), Sém. Congr., vol. 1, Soc. Math. France, pp. 531–559 (1996).

II. CONFERENCE TALKS :

- *12th International Conference on Discrete Mathematics: Discrete Geometry and Alexandrov surfaces*, Bucharest, September 2013.
- *Anniversary Conference Faculty of Sciences - 150 years*, Bucharest, August-September 2013.
- *Experimental and Theoretical Methods in Algebra, Geometry and Topology*, Eforie Nord, June 2013.
- *Colloque de géométries*, Mulhouse, September 2012.
- *The 10th International Workshop on Differential Geometry and its Applications*, Constanţa, August 2011.
- *11th International Conference on Discrete Mathematics: Covexity and Discrete Geometry*, Dortmund, July 2009.
- *7th Congress of Romanian Mathematicians*, Braşov, June-July 2011.
- *10th International Conference on Covexity and Discrete Mathematics*, Dortmund, August 2007.
- *6th Congress of Romanian Mathematicians*, Bucharest, June 2007.
- *Mathematical Society of Japan, 2005 Autumn Meeting*, Okayama, September 2005.
- *The 52nd Geometry Symposium of the Mathematical Society of Japan*, Fukuoka, August 2005.
- *9th International Conference on Discrete Mathematics and Embedded Structures*, Dortmund, August 2004.
- *Convexity and Discrete Mathematics*, Budapest, June 2004.
- *8th International Conference on Discrete Mathematics,* Dortmund, June 2000.
- *7th International Conference on Covexity and Discrete Mathematics*, Dortmund, June 1996.
- *1st Balkan Conference on Geometry*, Bucharest, September 1996.

Talks at workshops, seminars, etc.: Alicante (2011), Naha (2005), Nagoya (2005), Okayama (2005), Hokkaido Univ. (2005), Nara (2005), Kumamoto (2004, 2005, 2006), Timişoara (2004), Dortmund (2002), Bucharest (1997, 2011).

Reviewer for MathSciNet.