HARMONIC ANALYSIS ON PERTURBED CAYLEY TREES AND THE BOSE EINSTEIN CONDENSATION

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Abstract. We study some spectral properties of the adjacency operator of non homogeneous networks. The graphs under investigation are obtained by adding density zero perturbations to the homogeneous Cayley Trees. Apart from the natural mathematical meaning, such spectral properties are relevant for the Bose Einstein Condensation for the pure hopping model describing arrays of Josephson junctions on non homogeneous networks. The resulting topological model is described by a one particle Hamiltonian which is, up to an additive constant, the opposite of the adjacency operator on the graph. It is known that the Bose Einstein condensation already occurs for unperturbed homogeneous Cayley Trees. However, the particles condensate on the perturbed graph, even in the configuration space due to nonhomogeneity. Even if the graphs under consideration are exponentially growing, we show that it is enough to perturb in a negligible way the original graph in order to obtain a new network whose mathematical and physical properties dramatically change. Among the results proved in the present paper, we mention the following ones. The appearance of the *Hidden Spectrum* near the zero of the Hamiltonian, or equivalently below the norm of the adjacency. The latter is related to the value of the critical density and then with the appearance of the condensation phenomena. The investigation of the *recurrence/transience character* of the adjacency, which is connected to the possibility to construct locally normal states exhibiting the Bose Einstein condensation. Finally, the study of the *volume growth of the wave function* of the ground state of the Hamiltonian, which is nothing but the generalized Perron Frobenius eigenvector of the adjacency. This Perron Frobenius weight describes the spatial distribution of the condensate and its shape is connected with the possibility to construct locally normal states exhibiting the Bose Einstein condensation at a fixed density greater than the critical one.

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