

Turning Washington's heuristics in favor of Vandiver's conjecture

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Abstract

A famous conjecture bearing the name of Vandiver states that $h_p^+ = 1$ in the p -cyclotomic extension of \mathbb{Q} . Heuristics arguments of Washington, which have been briefly exposed in [La], p. 261 and [Wa], p. 158 suggest that the Vandiver conjecture should be false, if certain conditions of statistical independence are fulfilled. In this note we assume that Greenberg's conjecture is true for the p -th cyclotomic extensions and prove an elementary consequence of the assumption that Vandiver's conjecture fails for a certain value of p : the result indicates that there are deep correlations between this fact and the defect $\lambda^- i(p)$, where $i(p)$ is like usual the irregularity index of p , i.e. the number of Bernoulli numbers $B_{2k} \equiv 0 \pmod p$, $1 < k < (p-1)/2$. As a consequence, if one combines the various assumptions in Washington's heuristics, these turn, on base of the present result, into an argument in favor of the Vandiver's conjecture.

[La] S. Lang: Cyclotomic elds I and II, First Edition, Springer (1978,80)

[Wa] L. Washington: Introduction to cyclotomic elds, Springer, Graduate Texts in Mathematics 83, 2-nd edition (1996).