The Bing-Borsuk and the Busemann Conjectures

Dušan Repovš

Institute for Mathematics, Physics and Mechanics
Jadranska 19, P.O.Box 2965
University of Ljubljana
Ljubljana, Slovenia 1001
Email address: dusan.repovs@fmf.uni-lj.si
Homepage: pef.pef.uni-lj.si/∼dusanr/index.htm

We shall present a survey of two classical conjectures concerning the characterization of manifolds: the Bing Borsuk Conjecture asserts that every n-dimensional homogeneous ANR is a topological n-manifold, whereas the Busemann Conjecture asserts that every n-dimensional G-space is a topological n-manifold.

The key object in both cases are so-called *generalized manifolds*, i.e. ENR homology manifolds. We shall look at their history, from the early beginnings in 1930's to the present day, concentrating on those geometric properties of these spaces which are particular for dimensions 3 and 4, in comparison with generalized ($n > 4$)-manifolds.

In the second part of the talk we shall present the current state of the two conjectures (the work of Bing - Borsuk, Bestvina - Daverman - Venema - Walsh, Brahm, Bryant - Ferry - Mio - Weinberger, Busemann, Cannon, Daverman - Repovš, Daverman - Thickstun, Edwards, Halverson - Repovš, Krakus, Lacher - Repovš, Pedersen - Quinn - Ranicki, Thurston, and others). We shall also list open problems and related conjectures.