Stock Market Insider Trading in Continuous Time with Imperfect Dynamic Information

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Outline

1. Model Setup
2. Market Equilibrium
3. Examples

Related literature

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  - Insider's presence on the market increase informativeness of the price: Back (1992), Back and Pedersen (1998), Cho (2003), Campi and Çetin (2007)

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- Existence of unique markovian inconspicuous equilibrium pricing rule and characterization of the optimal insider's strategy when variance of the private signal is lower then variance of noisy demand at the end of trading period.
- Existence of non markovian inconspicuous equilibrium and characterization of optimal insider's strategies.

A Dynamic Information Model: Market structure

- Time horizon is finite: $[0,1]$
- Market consists of:
  - One riskless asset which has a constant growth rate $r = 0$.
  - A risky asset with final payoff $f(Z_1)$, where f is a strictly increasing function and Z_t, the time t profit of the firm which issued the stock, is given by

$$Z_t = v + \int_0^t \sigma_z(s) dB_s^1,$$

    where B_t^1 is a standard Brownian motion and v is $N(0,\sigma)$ independent of $\mathcal{F}_1^{B^1}$.
- P_t denotes the price of the risky asset at time t.

A Dynamic Information Model: Market participants

There are three types of agents on the market:

- Noisy/liquidity traders: their total demand by time t is given by standard Brownian motion B_t^2 independent of B^1, v
- Informed investor: observes $\mathcal{F}_t^I = \mathcal{F}_t^{B^1,B^2,v}$ and is risk-neutral, i.e. she solves

$$sup_\theta \mathbb{E}[X_1^\theta] = sup_\theta \mathbb{E}[(f(Z_1) - P_1)\theta_1 + \int_0^1 \theta_{s-} dP_s]$$

- Market maker: observes $\mathcal{F}_t^M = \mathcal{F}_t^Y$ where $Y_t = \theta_t + B_t^2$ is the total order process and sets the price according to

$$P_t = \mathbb{E}[f(Z_1)|\mathcal{F}_t^M]$$

Admissible Pricing Rule and Trading Strategy

Definition: *A pair of measurable functions, $w : [0,1] \to \mathbb{R}_+ \setminus \{0\}$ and $H \in C^{2,1}(\mathbb{R} \times [0,1])$, $H : \mathbb{R} \times [0,1] \to \mathbb{R}$, increasing in the first variable, is an admissible pricing rule ($(H, w) \in \mathcal{H}$) if:*

1. *The weighting function, $w(t)$, is given by $w(t) = \sum_{i=1}^{n} \alpha_i 1_{\{t \in (t_{i-1}, t_i]\}}$ and it satisfies $\int_0^1 w^2(t)dt = 1$.*
2. $\mathbb{E}\left[\int_0^1 H^2(\int_0^t w(s)dB_s^2, t)dt + H^2(\int_0^1 w(s)dB_s^2, 1)\right] < \infty$.
3. $\mathbb{E}\left(h_i^{-1}\left(\mathbb{E}\left[f(Z_1)|\mathcal{F}_{t_i}^Z\right]\right)\right)^2 < \infty$ *where* $h_i^{-1}(H(y, t_i)) = y$.

Moreover, (H, w) is a rational pricing rule if it satisfies

$$H(\int_0^t w(s)dY_s, t) = P(Y_{[0,t]}, t) = \mathbb{E}\left[f(Z_1)|\mathcal{F}_t^M\right].$$

Definition: *A strategy $\theta_t \in \mathcal{A}(H, w)$ if: it is $\mathcal{F}_t^I$-semimartingale,*

$$\mathbb{E}\left[\int_0^1 H^2\left(\int_0^t w(s)d\theta_{s-} + \int_0^t w(s)dB_s^2, t\right) dt\right] < \infty.$$

Moreover, the insider's trading strategy is inconspicuous if $Y_t = \theta_t + B_t^2$ is Brownian motion on its own filtration $\mathcal{F}_t^Y$.

Definition of Equilibrium

Definition *A pair* $((H^*, w^*), \theta^*)$ *is an equilibrium if* (H^*, w^*) *is an admissible pricing rule,* θ^* *is an admissible strategy, and:*

1. *Given* θ^**,* (H^*, w^*) *is a rational pricing rule, i.e. it satisfies*

$$H^*(\xi_t^*, t) = \mathbb{E}[f(Z_1)|\mathcal{F}_t^M]$$

   where $\xi_t^* = \int_0^t w^*(s)dY_s^*$

2. *Given* (H^*, w^*)*,* θ^* *solves the optimization problem*

$$sup_{\theta \in \mathcal{A}} \mathbb{E}[(f(Z_1) - H^*(\xi_1^*, 1))\theta_1 + \int_0^1 \theta_{s-} dH^*(\xi_s^*, s)]$$

Moreover, a pricing rule $(H^*(\xi, t), w^*(t))$ *is an inconspicuous equilibrium pricing rule if there exists an inconspicuous insider trading strategy* θ^* *such that* $((H^*, w^*), \theta^*)$ *is an equilibrium.*

Assumptions

I assume that the following conditions hold:

1. $\Sigma_z(t) = \int_0^t \sigma_z^2(s)ds < \infty$ thus can rescale: $\sigma^2 = 1 - \Sigma_z(1)$.
2. $F(Z_t, t) = \mathbb{E}\left[f(Z_1)|\mathcal{F}_t^Z\right]$, the fundamental value of the risky asset, is a square integrable martingale.
3. There exists an increasing admissible weighting function $g(t) = \sum_{i=1}^n \alpha_i 1_{\{t\in(t_{i-1},t_i]\}}$ such that:

$$\begin{aligned} \Delta(t) &> 0 \text{ for all } t \in [0,1]\backslash\{t_i\}_{i=1}^n \\ \Delta(t_i) &= 0 \text{ for all } t_i \\ \int_{t_{i-1}}^t \Delta^{-2}(s)ds &< \infty \text{ for all } t \in [t_{i-1}, t_i) \\ \lim_{t\to t_i} \int_{t_{i-1}}^t \Delta^{-1}(s)ds &= \infty. \end{aligned}$$

where $\Delta(t) = \Sigma_z(t) + \sigma^2 - \int_0^t g^2(s)ds$

Optimal Strategy

Proposition *Suppose that the assumptions are satisfied. Then, given an admissible pricing rule* $(H, w) \in \mathcal{H}$ *such that* $\int_0^t w^2(s)ds$ *is convex function,* $w(t) = \sum_{i=1}^n \alpha_i 1_{\{t \in (t_{i-1}, t_i]\}}$, *and* H *satisfies the partial differential equation*

$$H_t(y, t) + \frac{w^2(t)}{2} H_{yy}(y, t) = 0, \tag{1}$$

an admissible trading strategy $\theta^* \in \mathcal{A}(H, w)$ *is optimal for the insider if and only if:*

1. *The process* θ_t^* *is continuous and has bounded variation.*
2. *The weighted total order,* $\xi_t^* = \int_0^t w(s)d\theta_{s-}^* + \int_0^t w(s)dB_s^2$ *satisfies*
$$h_i\left(\xi_{t_i}^*\right) = H(\xi_{t_i}^*, t_i) = F\left(Z_{t_i}, t_i\right). \tag{2}$$

Sketch of the proof, $w(t) \equiv 1$ case

Consider a system of PDEs:

$$V_t + \frac{1}{2} V_{yy} + \frac{\sigma_z^2(t)}{2} V_{zz} = 0 \tag{3}$$

$$V_y + F(z,t) - H(y,t) = 0 \tag{4}$$

where $F(Z_t, t) = \mathbb{E}[f(Z_1)|\mathcal{F}_t^I]$.
If H satisfies (1) then there exists V - a solution of (3) and (4) such that for some y^* we will have for any $y, z \in \mathbb{R}$, $y \neq y^*$

$$V(y,z,1) > V(y^*,z,1) = 0$$

where $y^*(z)$ solves $H(y^*, 1) = f(z)$.
Since for any admissible trading strategy we have

$$\mathbb{E}[X_1^\theta] = \mathbb{E}[\int_0^1 (F(Z_s,s) - H(Y_{s-},s))d\theta_s + \int_0^1 \theta_{s-} dF + [\theta, F-H]_1]$$

Sketch of the proof

Then by applying Itô formula we get:

$$\begin{aligned}\mathbb{E}[X_1^\theta] &= \mathbb{E}[V(0,v,0) - V(Y_1, Z_1, 1) - \int_0^1 \frac{V_{yy}(Y_{s-}, Z_s, s)}{2} d\langle\theta^c\rangle_s \\ &+ \sum_{s\le 1} [\Delta V(Y_s, Z_s, s) - V_y(Y_s, Z_s, s)\Delta Y_s] \\ &+ \int_0^1 (F(Z_s, s) - H(Y_{s-}, s))dB_s^2]\end{aligned}$$

due to properties of V we are done

Existence of Equilibrium

Theorem *Suppose that the assumptions are satisfied. Then there exists an equilibrium and it is given by the weighting function*

$$w^*(s) = g(s),$$

the pricing rule

$$H^*(\xi, t) = \mathbb{E}\left[f\left(\xi + \int_t^1 g(s)dB_s^2\right)\right],$$

and the trading strategy θ_t^* *satisfying* $\theta_0^* = 0$ *and*

$$\begin{aligned} d\theta_t^* &= 1_{\{t\in(0,t_1]\}}\frac{(Z_t - \alpha_1 Y_t)\,\alpha_1}{\Sigma_z(t) + \sigma^2 - \int_0^t g^2(s)ds}dt \\ &+ \sum_{i=1}^{n-1} 1_{\{t\in(t_i,t_{i+1}]\}}\frac{(Z_t - Z_{t_i} - \alpha_{i+1}(Y_t - Y_{t_i}))\,\alpha_{i+1}}{\Sigma_z(t) + \sigma^2 - \int_0^t g^2(s)ds}dt \end{aligned}$$

Examples

Example 1: Let $\sigma = 1$ and $\sigma_z(t) = 0$, thus $\Sigma_z(t) + \sigma^2 - t = 1 - t$ is strictly positive on $[0, 1)$ and $\int_0^t \frac{ds}{(\Sigma_z(s)+\sigma^2-s)^2} = \frac{1}{1-t} < \infty$.
Therefore optimal informed trader's strategy is given by

$$d\theta_t^* = \frac{Z_1 - Y_t^*}{1 - t} dt$$

Example 2: Consider $\sigma = \sqrt{1 - \rho^2}$ and $\sigma_z(t) = \rho \in [0, 1)$ then $\Sigma_z(t) + \sigma^2 - t = (1 - \rho^2)(1 - t)$ is strictly positive on $[0, 1)$ and $\int_0^t \frac{ds}{(\Sigma_z(s)+\sigma^2-s)^2} = \frac{1}{(1-\rho^2)^2(1-t)} < \infty$.
Therefore optimal informed trader's strategy is given by

$$d\theta_t^* = \frac{Z_t - Y_t^*}{(1 - \rho^2)(1 - t)} dt$$

Example 2 ctd: limiting case $\rho \to 1$

Proposition *Consider a sequence of* $\rho_n \nearrow 1$ *and*

$$
\begin{aligned}
Y_t^n &= \int_0^t \frac{Z_s^n - Y_s^n}{(1-\rho_n^2)(1-s)} ds + B_t^2 \\
Z_t^n &= \sqrt{1-\rho_n^2} v + \rho_n B_t^1
\end{aligned}
$$

Then $Y^n - Z^n \to 0$ *weakly in Skorokhod topology.*

Proof Let $B_t^n = \frac{B_t^2 - \rho_n B_t^1}{\sqrt{1+\rho_n^2}}$ and consider

$$X_t^n = (1-t)^{\frac{1}{1-\rho_n^2}} \int_0^t (1-s)^{-\frac{1}{1-\rho_n^2}} dB_t^n$$

Since X is continuous and for any $\epsilon > 0$

$$\mathbb{E}|X_t^n - X_s^n|^{2m} \leq C_{2m}(\epsilon)|t-s|^m$$

for any $s, t \in [0, 1-\epsilon]$ and some constant $C_{2m}(\epsilon)$, we have that X^n is tight on $[0, 1)$. Since $X_1^n = 0$ for all n, X^n is tight on $[0, 1]$.

Work in Progress and Future Research

Work in Progress

1. Uniqueness of inconspicuous equilibrium pricing rule in the non markovian case.
2. Existence of an equilibrium when the market parameters do not satisfy the assumptions.
3. Extension of this model to include potential bankruptcy of the firm issuing the stock (with Campi and Çetin)

Future Extensions

1. More general utility functions.
2. Trading in stock <u>and</u> derivatives.