ABSTRACTS

Problems in estimating tax evasion and hidden economy

Lucian-Liviu Albu (speaker) and Andreea Iluzia Iacob

High tax rate and ineffective tax collection by fiscal authority are generally accepted by economists as main causes contributing to the rise of informal economy. Already the economists have established a relationship between tax rates and tax evasion or size of the informal economy. The higher is the level of taxation, the greater incentive is to participate in informal economic activities and escape taxes. At the macroeconomic level, there are a number of so-called indirect methods used to estimate the size and dynamics of informal economy ("Monetary Approach", "Implicit Labour Supply Method", "National Accountancy", "Energy Consumption Method", etc.) Unfortunately, many times there are huge differences among the estimated shares of informal or underground economy obtained by various methods. In our study, coming from general accepted finding of the theory in matter of modelling underground economy, and tax evasion, we concentrate on evaluating analytically the limit-values of certain important parameters involved in models used to estimate the size of tax evasion and underground economy and to explain the mechanisms of its dynamics. Thus, we are presenting a model, derived from the standard tax evasion model, by making endogenous the probability of detection in case of tax evasion. Then, we apply the obtained simulation model on available data. The second goal of the study is to report some conclusions on how much could be extended informal sector based on available data on labour supply in households and on the complex transmission mechanism from the tax policy decisions to the effective implication of agents into informal economy.

Malliavin calculus for jump proceses and applications

Vlad Bally

We develop an abstract version of the Malliavin Calculus which allows for example to compute numericaly sensitivietes. We focus on two specific problemes for which the standard calculus does not apply because of some singularities which come on in the problem. The first one concerns diferentation with respect to the jump times and the second one concerns diferentaition with respect to the amplitudes of the jumps in a model in which the coefficients have discontinuities.

On the completeness problem of bond market

Michal Baran (speaker) and Jerzy Zabczyk

The theory of bond market is not covered by classical market models since portfolios can contain infinite number of assets. Bonds are parametrized by $T \in [0, T^*]$, where $T^* < \infty$. The bond curve P is an element of some Banach space B and portfolio φ takes values in its dual B^*. The question is if any random variable H, from some space, can be represented as:

$$\int_0^{T^*} < \varphi, dP_{B^*,B} = H.$$

First results on bond market completeness come from the papers [1],[2]. We consider the problem of completeness with the use of bonds with dense and countable maturities in $[0, T^*]$ in the model given by the following equation for the forward rates:

$$df(t,T) = \alpha(t,T)dt + \sigma(t,T)dW(t) + \int_{\mathbb{R}} \gamma(t,T)xN(dt,dx).$$

Here, W is one dimensional Wiener process and N is a random Poisson measure. Problem of this type was first formulated in [3], where the result on the uniqueness of the martingale measure was established. In the talk we will formulate conditions which guarantie that the representation above holds or does not hold.

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On computer simulation in financial activities

Gheorghe Barbu

The economic decisional process becomes more and more complex due to the globalization which determines quick and many times unforeseeable changes.

Among others, the managers' mission is that of evaluating the risk they assume through the adoption of some decisions.

Taking decisions in economic processes depends on numerous factors such as prices instability (raw materials, materials, energy etc.), outlets' instability, competitors' strategy, inflation, State's monetary policy, economic stability of the respective economic zone, economic legislation, etc.

In order to estimate the effects of their decisions, managers should have at their disposal decisional variants scientifically substantiated.

Generally, the scientific study of an economic system or phenomenon may be done through real or artificial experimentation.

Real experimentation is difficult to achieve owing to the expenses and the risks it involves.

Because of the complexity of the real economic systems, of the stochastic dependencies between the different variables and the considered parameters, not all the systems can be adequately represented through a model that can be solved through analytical methods.

Although it does not offer accurate results, simulation permits the obtaining of information before the taking of a decision or the effective achievement of the respective system.

An interesting application can be the simulation of a business plan. In this field, there are a series of elements that are certain, but also others which are influenced by random causes or whose evolution pursues different laws of probability.

Through simulation one can obtain a series of variants, out of which, on the basis of a competent analysis, one can choose that one which to lead to the foreseen results.

The relaxed investor and the relaxed utility maximization problem

Sara Biagini (speaker) and Paolo Guasoni

For utility functions U finite only on the positive real line, Kramkov and Schachermayer showed that under a condition on U, the well- known Reasonable Asymptotic Elasticity, the associated utility maximization problem has a (unique) optimal solution, independently of the probabilistic model. What about the "relaxed" investor, whose utility does not satisfy RAE? This has been also addressed by Kramkov and Schachermayer, but the optimal solution is characterized only for sufficiently small initial endowments. Under a sufficient (and basically necessary) joint condition on the probabilistic model and the utility, we show by relaxation and duality techniques that the maximization problem admits solution for any initial endowment. However, a singular part may pop up, that is the optimal investment may have a component which is concentrated on a set of probability zero. This singular part may fail to be unique.

Utility maximization at random horizons: effect and time-consistency

Tahir Choulli (spaker) and Martin Schweizer

An investor is subject to many types of risk resulting from exiting the market before the closure. Every exit time planned or not has its specific risk and affect tremendously the optimal portfolio, however we have no idea how the optimal portfolio is altered with these kind of risks, and we do not know how serious the risk of each type of exit time, nor how to model it. Therefore, here we propose an approach to immunize the investor from risks that are related to a class of exit times that we describe in the paper. This class includes stopping times, planned exit times (like retirement, announced death,..., etc.), and many others. We prove that this immunization approach coincides with the stochastic time consistency of utility maximization problem. Stochastic time consistency is the economic concept of time-consistency using stopping time instead of deterministic time. We formalize and show the existence of the solution to the dual problem associated to this problem, and we state the duality formulation between the two problems. Under some reasonable assumptions of smoothness of the utility, and integrability conditions for the market model, we explicitly describe the solutions. In addition, we will present the key stochastic tools that are behind our current statements.

Local risk-minimization for defaultable markets

Alessandra Cretarola

We study the local risk-minimization approach for defaultable markets in a general setting where the asset price dynamics and the default time may influence each other. We consider a simple market model with two non-defaultable primitive assets (the money market account B and the discounted risky asset X) and a (discounted) defaultable claim H. Since it is impossible to hedge against the occurrence of a default by using a portfolio consisting only of the primitive assets, the market model extended with the defaultable claim is incomplete and the local risk-minimization approach appears as a reasonable choice in this context. Other quadratic hedging methods such as mean-variance hedging have been extensively studied in the context of defaultable markets by [1], [2], [3], [4] and [5]. The local risk-minimization method has been applied for the first time to the case of defaultable markets in [1], but only in the case where the default time and the underlying Brownian motion were independent. Here we consider the more general case where the dynamics of the risky assets may be influenced by the occurring of a default event and also the default time itself may depend on the assets prices behavior. In this general setting, we are able to find the Föllmer-Schweizer decomposition of a defaultable claim with random recovery rate and compute it explicitly in two particular cases, i.e. when default time depends on the risky asset behavior and when only a dependence of the discounted asset price on default time is occurring. References [1] Biagini F. and Cretarola A. (2007), Quadratic Hedging Methods for Defaultable Claims. Applied Mathematics and Optimization 56(3), 425- 443.

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Stock market insider trading in continuous time with imperfect dynamic information

Albina Danilova

We study the equilibrium pricing of asset shares in the presence of dynamic private information. The market consists of a risk-neutral informed agent who observes the firm value, noise traders, and competitive market makers who set share prices using the total order flow as a noisy signal of the insider's information. I provide a characterization of all optimal strategies, and prove existence of both markovian and non markovian equilibria by deriving closed form solutions for the optimal order process of the informed trader and the optimal pricing rule of the market maker. The consideration of non markovian equilibrium is relevant since market maker might decide to re-weight past information after recieving a new signal. Also, I show that a) there is a unique markovian equilibrium price process which allows the insider to trade undetected, and that b) the presence of an insider increases the market informational efficiency, in particular for times close to dividend payment.

Optimal consumption and portofolio for an insider in a market with jumps

Delphine David

We consider a stochastic optimal control problem in a financial market model with asymmetric information. In the market, we assume that there are two kinds of investors with different levels of information: a uninformed agent whose information coincides with the natural filtration of the price processes and an insider who has more information than the uninformed agent. Using forward integral techniques, we solve the optimal consumption and investment problem for the insider. We conclude by giving some examples.

Pricing and hedging asian basket spread options

Griselda Deelstra (speaker), Alexandre Petkovic and Michèle Vanmaele

Asian options, basket options and spread options have been extensively studied in literature. However, few papers deal with the problem of pricing general Asian basket spread options. This talk aims to fill this gap. In order to obtain prices and Greeks in a short computation time, we develop approximations formulae based on comonotonicity theory and moment matching methods. We compare their relative performances and explain how to choose the best approximation technique as a function of the Asian basket spread characteristics. We also give explicitly the Greeks for our proposed methods. In the last section we extend our results to options denominated in foreign currency. Key words Asian basket spread option, non-comonotonic sum, moment-matching, Shifted logextended skew normal law.

Advanced credit portfolio modeling and CDO pricing

Ernst Eberlein (speaker), Rüdiger Frey and Ernst August von Hammerstein

Modeling dependence is a key issue when one derives the loss distribution of a portfolio of credit instruments. We extend the factor model approach of Vasiček by using more sophisticated distributions for the factors. Completely different distributions from the class of generalized hyperbolic distributions and their limits can be chosen for the systematic and the idiosyncratic factor is this approach.

As a result an almost perfect fit to market quotes of DJ iTraxx Europe standard tranches is achieved. The correlation structure remains flat over all CDO tranches and maturities. No base correlation framework is needed.

Existence and uniqueness theory for the pricing equation in stochastic volatility models

Erik Ekström

We study the Black-Scholes equation in stochastic volatility models. In particular, we provide conditions under which the equation has a unique solution given by the risk-neutral expected value.

A method of moments approach to pricing double barrier options driven by Lévy processes

Bjorn Eriksson (speaker) and Martijn Pistorius

Barrier options are options that are (des-)activated when the underlying price process up-crosses and/or down-crosses certain pre-specified levels. Barrier options are amongst the most widely traded exotic options, which makes their valuation to an important question. A particular type of barrier options that is frequently traded in Foreign Exchange markets is the double barrier knock-out option. A common empirical observation is that financial returns data possesses heavy tails and excess kurtosis, features that cannot be captured by the geometric Brownian motion model. Successful modifications have been proposed to improve this standard model include stochastic volatility models and Lévy models (and combinations of both).

To price double barrier options under the geometric Brownian motion model Linetsky (2003) and Pellsner (1997) developed eigenfunction expansions and Ge man and Yor (1996) followed a Laplace transform approach. Both of the men tioned approaches draw on specific properties of the underlying driving process and cannot be readily extended to different settings. In a diffusion setting, Helmes et al. (2001) applied linear programming to calculate moments of exit distributions and Lasserre et al. (2006) studied pricing of European, Asian and single barrier options using semi-definite programming.

In this talk we will consider the pricing of double barrier options under the fairly general class of jump-diffusions whose generator maps polynomials to polynomials. The idea of the method is to rewrite the value of the derivative to be priced as a linear combination of moments of measures. Specifically the measures used are the exit location measure and the expected occupation measure. The exit location measure describes how the process ends, in the case of a Barrier option this would contain two parts. The distribution of the underlying at the exercise date given that it has not crossed the barrier and the distribu tion of crossing the barrier before the time of exercise. The expected occupation measure describes the behaviour of the process until it hits the domain of the exit location measure. In particular one of its moments is the integral of the underlying, this can be used to price certain Asian and Variance derivatives. The method allows the measures to be partitioned and we can use linear combinations of the moments of these partitioned measures to describe the payoff.

Following the description of the payoff we get to the core of the method, this is the fact that these measures can be related to each other using Dynkin's formula. By using monomials in Dynkin's formula and restricting ourself to jump-diffusions with a generator that maps polynomials to polynomials we gen erate a linear system of equations describing the relation between the moments of the measures involved. However solving this system will not necessarily result in moments of measures. To ensure this we also add moment conditions these will be either a linear system of equations or the requirement that certain matrices are positive semi-definite. Adding these parts together we get a linear or semi-definite programming problem. Each have their own dedicated solvers that can be used. We will illustrate the method with a number of numerical examples.

Probabilistic quantification of financial uncertainty

Hans Foellmer (speaker), Anne Gundel, Irina Penner and Walter Schachermayer

We discuss some recent advances in the probabilistic analysis of financial risk under model uncertainty, including risk measures and their dynamics, robust portfolio choice, and some asymptotic results involving large deviations.

About Malliavin Sobolev spaces and Besov spaces for Lévy processes

Christel Geiss (speaker) and Eija Laukkarinen

We consider a one-dimensional Lévy process (X_t) such that $S_t := e^{X_t}$ is a square integrable martingale. The Galtchouk-Kunita-Watanabe decomposition provides for any Borel function f such that $f(X_1) \in L_2$ a representation

$$f(S_1) = \mathbb{E}f(S_1) + \int_0^1 \theta_t dS_t + N,$$

where the random variable N is orthogonal to all stochastic integrals with respect to (S_t). Using specific chosen time nets $0 = t_0 < t_1 < \ldots < t_n = 1$ we investigate the relation between the rate of convergence r of

$$\left\| \int_0^1 \theta_t dS_t - \sum_{k=1}^{n} \theta_{t_{k-1}} (S_{t_k} - S_{t_{k-1}}) \right\|_{L_2} \leq cn^{-r}, \;\; n \to \infty$$

and a certain fractional smoothness in the Malliavin sense of $f(X_1)$.

Assuming (S_t) models a price process and f is a payoff function, the above expression stands for that part of the mean variance hedging error which is caused by discrete time hedging.

Optimization problem under liquidity risk with random trading times: regularity results and optimal strategies

Fausto Gozzi (speaker), Alessandra Cretarola, Huyen Xuan Pham and Peter Tankov

We study a mixed discrete/continuous time stochastic control problem arising from a portfolio/consumption choice problem in a market model with random trading times introduced in [1], and leading to a coupled system of integro-partial differential equations. Our main result is to derive smoothness C^1 results for the value functions. Consequently, we can prove the existence of the optimal control (portfolio/consumption strategy) that we characterize in feedback form in terms of the derivatives of the value functions.

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Consumption and investment with partial observation of stochastic volatility and economic conditions

Grzegorz Halaj

Model *à la* Merton was build but in discrete time and with stochastic volatility of prices that is not observed. The talk contributes to the literature on computational aspects of optimization with partial observation. It introduced new quantization approach to filtering - density quantization which reduces the original infinite dimensional state space of the problem to finite quantization set.

Wavelet FEM for option pricing under stochastic volatility

Norbert Hilber

We consider the numerical pricing of plain vanilla options by the Finite Element method for a fairly general class of stochastic volatility models.

The resulting pricing equations are (possibly) degenerate partial-integrodifferential equations (PIDEs) with non-constant coefficients. We study the variational formulation of these equations and introduce wavelet based sparse tensor product spaces to discretize them in (log-price) space.

We address the stabilization of PIDEs with non-negative characteristic form and apply it to the BNS model.

Finally, we present numerical examples, which include popular examples like the Heston or the Bates model.

Cumulative gains processes in finance and insurance

Lane Hughston

We consider a financial contract that delivers a single cash flow given by the terminal value of a cumulative gains process. The problem of modelling such an asset and associated derivatives is important, for example, in the determination of optimal insurance claims reserve policies, and in the pricing of reinsurance contracts. In the insurance setting, aggregate claims play the role of cumulative gains, and the terminal cash flow represents the totality of the claims payable for the given accounting period. A similar example arises when we consider the accumulation of losses in a credit portfolio, and value a contract that pays an amount equal to the totality of the losses over a given time interval. An expression for the value process of such an asset is derived as follows. We fix a probability space, together with a pricing measure, and model the terminal cash flow by a random variable; next, we model the cumulative gains process by the product of the terminal cash flow and an independent gamma bridge; finally, we take the filtration to be that generated by the cumulative gains

process. An explicit expression for the value process is obtained by taking the discounted expectation of the future cash flow, conditional on the relevant market information. The price of an Arrow-Debreu security on the cumulative gains process is determined, and is used to obtain a closed-form expression for the price of a European-style option on the value of the asset at the given intermediate time. The results obtained make use of remarkable properties of the gamma bridge process, and are applicable to a wide variety of financial products based on cumulative gains processes such as aggregate claims, credit portfolio losses, defined benefit pension schemes, emissions, and rainfall. (Co-authors: D. C. Brody, Imperial College London, and A. Macrina, King's College London and ETH Zurich. Downloadable at www.mth.kcl.ac.uk.)

On martingale optimality, BSDE and cross hedging

Peter Imkeller

A financial market model is considered on which agents (e.g. insurers) are subject to an exogenous financial risk, which they trade by issuing a risk bond. Typical risk sources are climate or weather. Buyers of the bond are able to invest in a market asset correlated with the exogenous risk. We investigate their utility maximization problem with respect to the correlation, and calculate bond prices using utility indifference. This hedging concept is interpreted by means of martingale optimality, and solved with BSDE tools. Prices are seen to decrease as a result of dynamic hedging. The increments are interpreted in terms of diversification pressure.

The variety and the optimal distributions with applications in economic publicity sistem

Gabriela Ionescu (speaker), Ion Purcaru and Ion Dobre

In this talk speaking about the optim structures of the publicity sistem then when have the best distribution of their elements in certain conditions of determination "the best distribution". In the project we used the right criterion of determination of the optimal distribution: "the principile of maximum variety of Gulasu", concordant which "we considerate that a distribution is optimal if it maximize the coeficient of diversity and is compatible with certain conditions".

On the combinatorics of iterated stochastic integrals

Farshid Jamshidian

This talk derives several identities for the iterated integrals of a general semimartingale. They involve powers, brackets, exponential and the stochastic exponential. Their form and derivations are combinatorial. The formulae simplify for continuous or finite-variation semimartingales, especially for counting processes. The results are motivated by chaotic representation of martingales, and a simple such application is given.

Pricing of defaultable rating-sensitive claims

Jacek Jakubowski

In my talk I present a formula for an ex-dividend price process S of defaultable rating-sensitive claim.

I consider processes on a complete probability space $(\Omega, \mathcal{F}, P)$. Denote by $\mathbb{F} = (\mathcal{F}_t)_{t \leq T}$ the reference filtration corresponding to observation of market without credit rating i.e. a filtration corresponding to the interest rate risk and other market factors that drives credit risk. By a defaultable rating-sensitive claim maturing at T we mean a set (X, A, Z, C, τ) where X is a $K-1$ dimensional vector of $\mathcal{F}_T$ measurable random variables, A is a $K-1$ dimensional vector valued stochastic process, $\mathbb{F}$ – adapted and of finite variation , Z is a $\mathbb{F}$ predictable $K-1$ dimensional vector valued process, C is a credit rating process with state space $\mathcal{K} = \{1, \ldots, K\}$ and τ is a default time.

A defaultable bond with fractional recovery of par value is a simple example of such claim. In this case bond's holder receives at maturity time T its face value provided that default time didn't occur before or at T. If default occurred before or at time T a recovery $\delta_{C_{\tau-}}$ is paid at the default time τ to the bond holders. The recovery payment depends on pre-default rating $C_{\tau-}$, and it is assumed that $\delta_i \in [0, 1)$ is a fixed number for each $i \in \mathcal{K} \setminus K$.

I define a special class of processes describing credit rating migration of defaultable instrument. It is a subclass of the class of conditional Markov chains and contains processes constructed by Lando and Bielecki, Rutkowski. A full characterization of this class will be given.

Then, a formula for an ex-dividend price process S of a defaultable rating-sensitive claim of general form will be presented. As an example, the closed formulae for ex-dividend prices of defaultable bonds with fractional recovery of par value and for credit sensitive notes will given.

Pricing and trading credit default swaps in a hazard process model

Monique Jeanblanc (speaker), Tomasz Bielecki and Marek Rutkowski

We study dynamics of the arbitrage prices of credit default swaps within a hazard process model of credit risk. We derive these dynamics without postulating that the immersion property is satisfied between some relevant filtrations. These results are then applied so to study the problem of replication of general defaultable claims, including some basket claims, by means of dynamic trading of credit default swaps.

Numerical analysis and computing of the Barles-Soner option pricing model

Rafael Company, Lucas Jodar (speaker) and Juan R. Pintos

This talk deals with the numerical analysis and computing of a nonlinear Black-Scholes equation modeling transaction costs arising in the hedging of portfolios. The equation is associated to the Barles-Soner model and involves a correction volatility function Ψ whose exact expression is not known. A fourth order numerical method based on the semidiscretization technique is proposed. By using an implkit expression of Ψ previously obtained, consistency and stability of the numerical solution are studied. Illustrative examples for European call options are included.

Implied integrated volatility

Ruth Kaila

Our aim is to introduce the new concept of integrated volatility implied by option prices, called the *implied integrated volatility*, and to present how this quantity can be estimated using statistical inverse methods.

The integrated volatility is the time-average of the stock price variance. It is a fundamental quantity related to the stock price returns as these returns depend on the stock price volatility only via the integrated volatility. The implied integrated volatility is this quantity implied by option market prices. In a general level, the implied integrated volatility is an extension to the commonly known Black-Scholes implied volatility.

The Black-Scholes implied volatility is based on the assumption of constant stock price volatility. This assumption is non-tenable which is reflected as a volatility smile, an inconsistency of volatilities implied by option prices with different strikes. In contrast, the integrated volatility is based on the assumption that stock price volatility is a stochastic process. This volatility is independent of the option strike price.

In 1987, Hull and White showed how the price of a European option with stochastic volatility, denoted by $U^{HW}(t, X_t; K, T; \bar{\sigma}^2)$, can be given as the expectation of Black-Scholes prices integrated over the distribution of the integrated volatility, that is

$$U^{HW}(t, X_t; K, T; \sigma_t^2) = \int U^{BS}(t, X_t; K, T; \bar{\sigma}_t^2 \mid \sigma_t^2) d\bar{\sigma}_t^2, \tag{1}$$

where U^{BS} is the commonly known Black-Scholes option price, X_t is the stock price, K is the strike price, T is the maturity, σ_t^2 is the instantaneous variance, $\bar{\sigma}_t^2$ is the integrated volatility, given by

$$\bar{\sigma}_t^2 = \frac{1}{T-t} \int_t^T \sigma_s^2 ds, \quad 0 \leq t < T, \tag{2}$$

and $\pi(\bar{\sigma}_t^2 \mid \sigma_t^2)$ is the conditional distribution of $\bar{\sigma}_t^2$ given σ_t^2.

The Hull-White pricing formula (1), which is the bedrock of our work, gives option prices as functionals of the distribution of the integrated volatility. Our central idea is that if option

market prices coincide with the corresponding Hull-White prices, this formula can be used to extract information on the integrated volatility implied by option market prices.

We assume that observed option prices are given by the Hull-White formula and estimate the distribution of the integrated volatility implied by these prices. Using a Bayesian approach to this ill-posed inverse problem we first calculate a Maximum-A-Priori estimate for this volatility and then study the reliability of this estimate with Markov Chain Monte Carlo methods.

We suggest that the implied integrated volatility can be used in volatility estimation, in pricing illiquid options consistently with corresponding liquid ones, and in hedging options. In addition to a point estimate, the Bayesian approach provides information on the reliability of these estimates.

Moment explosions and long-term behaviour of affine stochastic volatility models

Martin Keller-Ressel

We consider a class of asset pricing models, where the risk-neutral joint process of log-price and its stochastic variance is given by an affine process in the sense of Duffie, Filipovic, and Schachermayer. First we obtain conditions for the price process to be conservative and a martingale. Then we present results on the long-term behavior of the model, including an expression for the invariant distribution of the stochastic variance process. We study moment explosions of the price process, and provide explicit expressions for the time at which a moment of given order becomes infinite. We discuss applications of these results, in particular to the asymptotics of the implied volatility smile, and conclude with some explicit calculations for the Heston model without and with added jumps, a model of Bates and the Barndorff-Nielsen-Shephard model.

Operational risk, Pareto copulas and regular variation

Claudia Klüppelberg (speaker), Klaus Böcker, Yuliya Bregman and Irmingard Eder

Simultaneous losses occur for instance in operational risk or in insurance risk processes in different event type/business line cells.

To model the dependence structure of such risk processes we invoke the concept of Pareto Levy copulas. Similarly as distributional copulas separate the marginal distributions and the dependence structure, Levy copulas separate the marginal Levy processes and the dependence structure.

Specific Levy copulas interpolating between independence and complete dependence of Levy processes provide scenarios to assess the influence of dependence on appropriate risk measures.

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Quantile hedging for an insider

Przemyslaw Klusik (speaker) and Zbigniew Palmowski

In a complete financial market every contingent claim can be mimicked by portfolio consisting of underlying assets. The price of it has to be equal to the cost of building such replicating portfolio. An insurance company selling investment products usually doesn't get enough money for perfect hedging. We show an optimal hedging strategy if an extra information is used.

On divergence utilities

Michael Kupper (speaker) and Alexander Cherny

We investigate a class of concave monetary utility functions, which we call divergence utilities. Divergence utilities can be viewed as the translation invariant hull of classical expected utilities. This class is rather wide and includes, in particular, the entropic utility. More important, this class is very convenient analytically. We provide several representations, an explicit solution of the portfolio optimization problem as well as an explicit solution of the risk sharing problem.

On weak Kyle-Back models for the max and argmax

Arturo Kohatsu-Higa

We introduce a weak Kyle-Back type model for equilibrium in markets with insiders where the information possessed by the insider is a functional of the demand. Finally we compare the cases of the max and argmax of the demand.

Monte Carlo Greeks for financial products via approximative transitional densities.

Jörg Kampen, Anastasia Kolodko (speaker) and John Schoenmakers

We introduce efficient Monte Carlo estimators for the valuation of high-dimensional derivatives and their sensitivities ("Greeks"). These estimators are based on an analytical, usually approximative representation of the underlying density. We study approximative densities obtained by the WKB method. The results are applied in the context of a Libor market model.

Nonlinear stochastic integration theory with applications to price-impact models

Christoph Kühn

In this talk we consider nonlinear stochastic integrals of Itô-type w.r.t. a family of semimartingales which depend on a spatial parameter. These integrals were introduced by Kunita (1990) and Carmona and Nualart (1990).The extension of the elementary nonlinear integral is based on the condition that the semimartingale kernel has nice continuity properties in the spatial parameter. We are interested in the case that no continuity is available and discuss different directions of generalization. This brings us beyond the case that any integral can be approximated by integrals with integrands taking only finitely many values.

Furthermore we discuss an application to the utility maximization problem of a large investor whose trades move the market price of an illiquid asset.

Some remarks on American option prices in exponential Lévy models

Damien Lamberton (speaker) and Mohammed Mikou

We study American option prices in exponential Lévy models. In particular, we prove the continuity of the exercise boundary and recover and extend Levendorvskii's results on the behavior of the critical price near maturity.

Optimal switching over multiple regimes

Huyên Pham, Vathana Ly Vath (speaker) and Xun Yu Zhou

This talk presents the optimal switching problem for a general one-dimensional diffusion with multiple (more than two) regimes. This is motivated in the real options literature by the investment problem of a firm managing several production modes while facing uncertainties. A viscosity solutions approach is employed to carry out a fine analysis on the associated system of variational inequalities, leading to sharp qualitative characterizations of the switching regions. These characterizations, in turn, reduce the switching problem into one of finding

a finite number of threshold values in state that would trigger switchings. The results of our analysis take several qualitatively different forms depending on model parameters, and the issue of when and where it is optimal to switch is addressed. The general results are then demonstrated by the three-regime case, where a quasi-explicit solution is obtained, and a numerical procedure to find these critical values is devised in terms of the expectation functionals of hitting times for one-dimensional diffusions.

Local time and the pricing of time-dependent barrier options

Aleksandar Mijatovic

A time-dependent barrier option is a derivative security that delivers the terminal payoff at expiry T if neither of the continuous time-dependent barriers $b, B : [0, T] \to R$ (satisfying $b(t) < B(t)$ for all t) have been hit during the time interval $[0, T]$. In this talk we describe a decomposition of the time-dependent barrier option price into the corresponding European option price minus the barrier premium for a wide class of linear diffusions, possibly discontinuous payoff functions and twice differentiable barrier functions b, B. We show that the barrier premium can be expressed as an integral of the option's delta at the barriers and that the pair of functions describing the deltas at the barriers solves a system of Volterra integral equations of the first kind.

Copula-based martingale processes and financial prices dynamics

Umberto Cherubini, Sabrina Mulinacci (speaker) and Silvia Romagnoli

We suggest a new technique to construct Markov processes by means of products of copula functions, in the spirit of Darsow et al,(1992). The approach requires to define: i) a sequence of distribution functions of the increments of the process; ii)a sequence of copula functions representing dependence between each increment of the process and the corresponding level of the process before the increment. We show that the approach is well suited to impose restrictions that may ensure the process to be a martingale. More precisely, we single out two classes of Markov processes endowed with the martingale condition: i) processes with independent increments with zero mean distributions; ii) processes with symmetric increments linked to the initial levels by a symmetric copula. As most of the current financial mathematics literature is limited to the independent increments class, we show that copula functions may be used to produce new martingale processes within this class, and many more in the new class of processes with symmetric increments.

Time-inconsistent control

Agatha Murgoci (speaker) and Tomas Bjork

We are studying optimization problems of the type:

$$\max_u E_t[F(X_T)] + G(E_t[X_T])$$

where X_t is some stochastic process. This type of problem is time- inconsistent and cannot be solved by traditional tools of dynamic programming. This is why we take a game theoretic approach and consider the optimal strategy from a sub-perfect Nash equilibrium. We view the problem as a game where, at each point in time t, we have one separate player no t. Player t choses his/her strategy $u(t; X_t)$ taking into account the optimal strategies of the following players $\hat{u}(s; X_s)$, $s \leq t$. We prove that this optimality criteria is leading to a system of PDE-s similar to the classic Hamilton-Jacobi-Bellman equation, with an embedded fixed point problem. We find the adjusted HJB system of equations both in discrete and continuous time.

Applications to the above problem include portfolio allocation with a multi-period mean variance preferences and various hedging prolems. We solve for several specific examples, such as mean-variance portfolio optimization where the underlying asset has jumps, and obtain analytical solutions.

Hedging of defaultable rating-sensitive claims

Mariusz Niewęgłowski

The problem of hedging of defaultable claims have been recently studied by Bielecki et al [1,2]. We address the issue of hedging of defaultable rating-sensitive claims which, to our knowledge was not yet studied. Assuming that rating process follows a doubly stochastic Markov Chain (Jakubowski, Niewęgłowski [4]) we establish suitable representation theorem. This theorem leads to the self-financing portfolio that replicates rating sensitive claims. In particulary we will consider claims that are depending on the rating at maturity and claims made at default time that are depending on the credit rating before default time. We also give some results on the replication of the generic defaultable rating sensitive claims. The results we present, can be viewed as the generalization of the work by Blanchet-Scalliet and Jeanblanc [3] to the credit migration environment.

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Minimal-variance hedging in large financial markets: random fields approach

Giulia Di Nunno (speaker) and Inga B. Eide.

Inspired by the recent literature on hedging in large financial markets, i.e. markets with a countable number of securities available, we focus our attention on the problem of minimal-variance hedging. Our approach uses techniques of stochastic calculus on random fields: in particular we introduce the martingale random fields with respect to which an Ito type stochastic calculus and a non-anticipating stochastic derivation can be introduced. The stochastic derivative turns out to be directly linked with the minimal variance hedging strategy in a risk-neutral framework. The framework suggested allows extension to a possible large financial market model where a continuum of securities is available.

A Malliavin calculus approach to a general maximum principle for stochastic control of jump diffusions

Bernt Oksendal (speaker) and Xunyu Zhou

The classical maximum principle for optimal control of solutions of stochastic differential equations (developed by Pontryagin (deterministic case), Bismut, Bensoussan, Haussmann and others), assumes that the system is Markovian and that the controller has access to full, updated information about the system at all times. The classical solution method involves an adjoint process defined as the solution of a backward stochastic differential equation, which is often difficult to solve.

We apply Malliavin calculus for Lévy processes to obtain a generalized maximum principle valid for non-Markovian systems and with (possibly) only partial information available for the controller. The backward stochastic differential equation is replaced by expressions involving the Malliavin derivatives of the quantities of the system. The results are illustrated by some applications to finance.

Existence and uniqueness of solutions for some degenerate valuation equations and applications

Marco Papi

In finance, a stochastic model usually describes a random variable which can take its value on an infinite domain. Using such a model, many problems of financial derivative pricing can be written as degenerate parabolic problems defined on an infinite domain. One usual way to obtain numerical solutions of such problems is to solve the problem on a large finite domain. Since parabolic problems on a finite domain need boundary conditions, some approximate boundary conditions on artificial boundaries need to be added. In this talk we show that, under a condition ensuring that the stochastic process associated with the differential operator never leaves the domain, the degenerate parabolic problem has one and only one continuous viscosity solution, where no boundary condition is specified. In particular, the solution is obtained by studying some approximating problems. In addition, we allow the diffusion conefficient to be simultaneously structurally degenerate and, say, Holder continuous at the boundary. The study can be applicable to degenerate valuation equations involving for instance interest rates and stochastic volatility, in order to obtain an accurate and computationally efficient numerical scheme without using any artificial conditions. Numerical experiments show that if the singularity-separating method and extrapolation techniques are used, then a substantial improvement in the approximation can be obtained.

Dual problems in financial optimization

Teemu Pennanen

Duality theory of convex optimization is concerned with the study of pairs of convex optimization problems whose solutions and optimal values are closely related to each other. One of the earliest instances is the duality theory of linear programming developed in the 1940's, while the general theory matured in the 1960's. The duality theory was specialized to optimization of (discrete time) stochastic processes in the 1970's. In this talk, we specialize the results further to certain problems in financial optimization and find some interesting connections to recent developments in financial mathematics. In particular, we obtain generalizations of the duality theory for optimal stopping developed by Davis and Karatzas and applied by Rogers in numerical valuation of American options. Another instance is portfolio optimization in the currency market model of Kabanov for which we find a natural dual problem with computational advantages similar to the technique of Rogers.

Currency markets with convex transaction costs

Irina Penner (speaker) and Teemu Pennanen

We characterize no arbitrage criteria and hedging of claim processes in a discrete-time financial market model with convex transaction costs. Our setting applies to Kabanov's modeling of currency markets with proportional transaction costs but allows more general to include non-linear illiquidity effects. We introduce the "robust no scalable arbitrage property", a notion that extends the robust no arbitrage property introduced by Schachermayer. Our main theorem states that the set of all hedgeable claims is closed with respect to convergence in measure if the market has the robust no scalable arbitrage property. This result allows to provide a dual characterization of premium processes that are sufficient to super-replicate a given claim process under convex transaction costs. We also extend the fundamental theorem of asset pricing to our setting.

Analysis and evaluation of risk

Doina Perpelea

Mathematical modeling is a technique according to which the system is associated to a proper model which stands for all logical interactions of the components of the system and their changing mechanism in time. With the help of the implemented algorithm a succession of states is provoked to the model and to the system, starting from an initial state given and considered to he known. This one can determine the evolution of the system and the validity of the used model in direct relationship with the topic which is going to be approached.

The concept of risk is associated with the action of exposing to a changer created by an unexpected event (hazard). The evaluation of the risk studies the distribution of the probability of the hazard to appear, thus being associated with the consequences of such an appearance.

The process of evaluation of the risk involves complex relationship between the scientist, technologist the one who decides and society.

The process of development and application of mathematical models in the analysis and evaluation of industrial risk is tightly connected with the behavior of the system, the accuracy of this information, the degree of sensitivity to the risk of the one who decides the ability to assess correctly the consequences of the appearance of a hazard.

In this talk I present different models to determine the characteristics of reliability of the components of the characteristics of reliability of the minimum set of cuttings. After obtaining the characteristics of the minimum sets of cuttings one can determine the characteristics of reliability of the whole system (installation).

The example presented above illustrates / shows the way of application of these models. A faulty tree is reduced by Boleen diminution to the minimum set of cuttings, followed by the determination of the parameters of the components and then the parameters of the minimum sets of cuttings and finally the parameters of the whole system.

The law of the minimal price

Eckhard Platen

The talk introduces a general market setting under which the Law of One Price does no longer hold. Instead the Law of the Minimal Price will be derived, which for a range of contingent claims provides lower prices than suggested under the currently prevailing approach. This new law only requires the existence of the numeraire portfolio, which turns out to be the portfolio that maximizes expected logarithmic utility. In several ways the numeraire portfolio cannot be outperformed by any nonnegative portfolio. The new Law of the Minimal Price leads directly to the real world pricing formula, which uses the numeraire portfolio as numeraire and the real world probability measure as pricing measure when computing conditional expectations. The pricing and hedging of extreme maturity bonds illustrates that the price of a zero coupon bond, when obtained under the Law of the Minimal Price, can be far less expensive than when calculated under the risk neutral approach.

The impact of different points of view in "Malliavin's calculus" for jump processes

Maurizio Pratelli

The aim of this talk is to show how different approaches to the "stochastic calculus of variations", which coincide in the case of Wiener Process, give rise to different definitions and results in the case of Poisson process and more generally in the case of Jump Processes. The properties resulting from these different definitions are compared, with emphasis on numeric consequences in problems related to Mathematical Finance.

How to view arbitrage in the presence of friction?

Miklos Rasonyi (speaker), Paolo Guasoni and Walter Schachermayer

We present some new developments concerning the fundamental theorem of asset pricing under proportional transaction costs. We propose a new, more stringent notion of arbitrage for discrete-time market models and highlight its advantages.

We investigate what happens if we postulate absence of arbitrage for arbitrarily small transaction costs (previous results show that this is a condition typically satisfied). For models with continuous trajectories it turns out that the existence of consistent price systems (analogues of martingale measures) follows from the absence of arbitrage only (no need for no-free-lunch type conditions).

Building on a new, economically meaningful notion of admissibility we manage to prove a general version of the fundamental theorem of asset pricing for processes with jumps, too.

Numerical derivative pricing in multidimensional non-BS models

Nils Reich (speaker), Norbert Hilber, Christoph Schwab and Cristoph Winter

We present finite element methods for derivatives pricing in general market models. Admissible processes are strong Markov processes, possibly non-stationary or time inhomogeneous, of jump-diffusion and pure jump type, including in particular Lévy and additive processes. Multidimensional Lévy models with non-trivial dependence among the jumps are also considered in detail.

Our approach is based on stabilized Galerkin discretization of the process' infinitesimal generator resp. its Dirichlet form in a wavelet basis. The methods provide a unified methodology to analyze single and multiperiod contracts of European, American or exotic style, on single underlyings or on baskets.

We address the fast extraction of Greeks and other model sensitivities. Numerical analysis in the domains of the infinitesimal generators of the price processes is described. Finally, numerical examples are presented. Joint work of the CMQF group in the Seminar for Applied Mathematics, ETH Zurich.

Exponential utility maximization under partial information

Marina Santacroce (speaker) and Michael Mania

In this talk I will consider the exponential utility maximization problem under partial information. The underlying asset price process follows a continuous semimartingale and strategies have to be constructed when only part of the information in the market is available. I will show that this problem is equivalent to a new exponential optimization problem, which is formulated in terms of the observable processes. The value process of the reduced problem is shown to be the unique solution of a backward stochastic differential equation (BSDE), which characterizes the optimal strategy. Two particular cases of diffusion market models, for which an explicit solution has been provided, will be examined.

Optimal portfolio liquidation for risk-averse investors

Alexander Schied (speaker) and Torsten Schoeneborn

A variety of circumstances can force a market participant to liquidate an asset position that is so large that selling it will significantly impact the underlying asset price. In this talk, we will discuss the problem of constructing optimized liquidation algorithms that maximize the expected utility of the seller. When setting up this problem as a stochastic control problem, a finite-time liquidation requirement acts as a finite-fuel constraint, and the corresponding HJB equation becomes singular. We show how it can be solved in the CARA utility case by variational techniques. General utility functions can be handled in the case of an infinite time

horizon. We also analyze the sensitivities of the value function and the optimal strategy with respect to the various model parameters. In particular, we find that the optimal strategy is aggressive or passive in-the-money, respectively, if and only if the utility function displays increasing or decreasing risk aversion. Surprisingly, only few further monotonicity relations exist with respect to the other parameters. We point out in particular that the speed by which the remaining asset position is sold can be decreasing in the size of the position but increasing in the liquidity price impact.

A generalization of Panjer's recursion and numerically stable risk aggregation

Uwe Schmock (speaker), Stefan Gerhold and Richard Warnung

Portfolio credit risk models as well as models for operational risk can often be treated analogously to the collective risk model coming from insurance. Applying the classical Panjer recursion in the collective risk model can lead to numerical instabilities, for instance if the claim number distribution is extended negative binomial or extended logarithmic. We present a generalization of Panjer's recursion that leads to numerically stable algorithms. The algorithm can be applied to the collective risk model, where the claim number follows, for example, a Poisson distribution mixed over a tempered stable distribution with exponent in (0,1). DePril's recursion can be generalized in the same vein. Time permitting, we also mention an analogue of our method for the collective model with a severity distribution having mixed support.

Super-replication in an illiquid model

Mete Soner

For a given contingent claim $g(S_T)$ and a stock price process $S_\cdot$, we consider the minimal super-replication price

$$v(t,s) := \{z \ : \ \exists Y_\cdot \in A_{t,s} \ s.t. \ Z_T^{t,s,z,Y} \geq g(S_T^{t,s}) \ a.s., \ \},$$

where as usual

$$dS_u = S_u \sigma dW_u,$$

the admissible set $A_{t,s}$ is given in [1], and the "wealth" process Z is given by

$$Z_T^{t,s,z,Y} = z + \int_t^T Y_u dS_u - \int_0^T \frac{1}{4\ell(S_u)} \, d[Y,Y]_u^c,$$

and the function $\ell(s)0$ is a measure of liquidity of the market. This function is defined in the model of Cetin-Jarrow-Protter. Under our assumptions, admissible portfolio processes satisfy

$$dY_u^c = \alpha_u \, du + \Gamma_u \, dS_u.$$

Hence, $[Y,Y]^c_u = \sigma^2 S_u^2 \Gamma_u^2 \, du$. We prove that the minimal price is the unique solution of

$$-v_t + \sup_{B \geq 0}\{-\frac{1}{2}\sigma^2 s^2 (v_{ss} + B) - \frac{1}{\ell}\sigma^2 s^2 (v_{ss} + B)^2$$

on $t < T$ with terminal data $v(T,s) = g(s)$. In particular, there is liquidity premium.

Multivariate Feller conditions in discrete time term structure models

Peter Spreij (speaker), Peter Vlaar and Enno Veerman

The relevance of the Feller conditions for discrete time macro-finance affine term structure models is investigated. The Feller conditions are usually imposed on a continuous time multivariate square root process to ensure that the roots have nonnegative arguments. For a discrete time approximate model, the Feller conditions loose their significance. Since the noise involves standard normal errors, there is always a positive probability that arguments of square roots become negative. Nevertheless, it is not uncommon to impose the Feller conditions, whereas it has also been observed that even without the Feller conditions imposed, for a practically relevant term structure model, negative arguments rarely occur.

We investigate the relevance of imposing the Feller conditions for a macro-finance two-factor affine term structure model, where the factors (ex-ante real short term interest rate and expected inflation) are modelled as a square root process.

Three different submodels will be estimated, using about 200 quarterly German data points, either with or without the Feller conditions on the parameters. For the six resulting cases we have compared the yields, that are either obtained by (approximate) analytic exponentially affine expressions or those obtained through Monte Carlo simulations of very high numbers of sample paths. It turned out that the differences were rarely statistically significant, and never economically relevant, as they were always below one basis point.

Long run GOP and risk sensitive GOP under proportional transaction costs

Lukasz Stettner

We consider a general continuous time market model with proportional transaction costs that consists of fixed proportional part (so called managing cost) and the value proportional to the volume of transaction. The asset prices are geometric Levy processes with parameters depending on economic factors. We maximize long run portfolio growth or risk sensitive portfolio growth. It can be shown that these problems correspond to impulsive control problems of portions of wealth invested in assets with average per unit time functional or ergodic risk sensitive functional respectively. We impose an obligatory diversification of portfolio i.e. an obligatory transaction when the process of portions of wealth invested in assets is close to the boundary of the simplex (the set of portfolios). Exhttp://fr.yahoo.com/istence of continuous solutions to suitable Bellman equation and the form of optimal strategies are then shown.

Foreign exchange rates and foreign exchange derivatives: an optimal portfolio based theory and empirics

Srdjan Stojanovic

We establish a general, optimal portfolio based theory for foreign exchange rates, in the context of multidimensional, possibly incomplete, Ito SDE market/econometric models. In such a theory the market risk aversion is a very important parameter for the dynamics of foreign exchange rates. Furthermore, we establish a consistent and as general theory of foreign exchange derivatives. Some examples and the consistency of the introduced theory with the empirical observations are discussed as well. In particular, we explain why Japanese Yen falls when investors ignore risks.

Sub-fractional Brownian motion as a model in finance

Constantin Tudor

We consider an extension of the Brownian motion, called sub-fractional Brownian motion (sfBm for short), which has properties analogous to those of the fractional Brownian motion (it has self-similar, long-range dependence and Hĺolder paths and it is neither a Markov process nor a semimartingale). Moreover sfBm has non-stationary increments and the increments over non-overlapping intervals are more weakly correlated and their covariance decays at a higher rate in comparison with fBm (for this reason is called sfBm). The multiple sub-fractional integrals are introduced by using a transfer idea from multiple Wiener-Itô integrals via an operator involving Erdély-Kober fractional integrals. The chaos form of the corresponding anticipating integral is considered. A sub-fractional Girsanov theorem and a Clark-Ocone formula are presented. As an application we develop option pricing in a sub-fractional Black-Scholes market.

Regularity of solutions to the term structure equation

Johan Tysk (speaker) and Erik Ekström

We prove, under very general conditions on the coefficients, regularity of solutions to the term structure equation up to and including the lateral boundary. This enables us to find the appropriate boundary conditions covering both the case of models that allow the short rate to become zero and also models that predict positive rates. We also prove uniqueness under these conditions. Furthermore, we investigate the case of models allowing negative rates.

On the absolute continuity and singularity of measures: separating times

Mikhail Urusov (speaker) and Alexander Cherny

We introduce the notion of separating time for a pair of measures P and $\tilde{P}$ on a filtered space, which describes the mutual arrangement of P and $\tilde{P}$ from the viewpoint of their absolute continuity and singularity. Furthermore, we find the explicit form of the separating time in the case, where P and $\tilde{P}$ are distributions of one-dimensional diffusions. As a consequence, we obtain deterministic criteria for the (local) absolute continuity and singularity of P and $\tilde{P}$ in terms of the diffusion coefficients. On the one hand, the questions of such type arise in Mathematical Finance in connection with different variants of the Fundamental Theorem of Asset Pricing (see e.g. Harrison and Kreps (1979), Harrison and Pliska (1981), Dalang, Morton, and Willinger (1990), Delbaen and Schachermayer (1994), Delbaen and Schachermayer (1998), Cherny (2007), ...), where one needs to find an equivalent probability under which the discounted prices are martingales (local martingales, sigma-martingales, ...). On the other hand, the questions of absolute continuity and singularity of measures are instructive in the Stochastic Portfolio Theory that was introduced in Fernholz (1999) (see also the monograph Fernholz (2002)), where it turns out that we have relative arbitrage and no equivalent local martingale measure in diverse markets (see e.g. Fernholz, Karatzas, and Kardaras (2005)). Nevertheless, the concept of diverse market is realistic and mathematically tractable as is shown in Fernholz, Karatzas, and Kardaras (2005) and in Fernholz and Karatzas (2008). Furthermore, studying the questions of absolute continuity and singularity of measures can also contribute to the Stochastic Portfolio Theory. For instance, Osterrieder and Rheinĺander (2006) construct a variety of diverse markets via an absolutely continuous but non-equivalent measure change.

We introduce the notion of separating time and we find the explicit form of the separating time for one-dimensional diffusions. These and some other results are presented in Cherny and Urusov (2006). We also show how they can be applied in the study of no-arbitrage.

Robust replication under model uncertainty

Esko Valkeila (speaker), Pavel Gapeev and Tommi Sottinen

We consider the robust hedging problem in which an investor wants to super-hedge an option in the framework of uncertainty in a model of a stock price process. We argue that in order to super-replicate European options with convex payoff, the investor must assume that the market is efficient.

Asymptotic behaviour and gradient representation for cad-lag solutions of S.D.E.

Bogdan Iftimie, Ionel Molnar and Constantin Vârsan (speaker)

Piece-wise continuous (cad-lag) solutions of s.d.e. driven by nonlinear vector fields and containing jumps are studied involving the gradient reprensentation of the jump dynamic part augmented with a sequence of boundary problems.

Double-sided Parisian options and sojourns

J.A.M. van der Weide

A Parisian option is a special type of barrier option. A down-and-in call barrier option with strike K, barrier L and time to maturity T pays only if the asset price $S(t)$ hits the barrier before maturity in which case the pay-off is given by $(S(T) - K)^+$, otherwise it expires worthless. There exist also down-and-out, up-and-in and up-and-out barrier options. A down-and-in Parisian option is a barrier option that only knocks in if the asset price is below the barrier during a given unbroken time period of length D. A down-and-in double-sided Parisian option is defined by two barriers L_1 and L_2, $L_2 < L_1$, and two time lengths D_1 and D_2. The option knocks in if either the asset price is above L_1 during an unbroken time period D_1 or below L_2 during an unbroken time period D_2. Modeling the asset price as a geometric Brownian motion, the Laplace transform of the arbitrage-free price of the option was calculated in the PhD thesis of Jasper Anderluh. An important step in the derivation of this Laplace transform is a formula for the Laplace transform of the first time at which Brownian motion is a time interval of length D_1 above some level l_1 or during a time interval of length D_2 below some level $-l_2$, where $l_1, l_2 \geq 0$. One of the consequences of this formula is a nice formula for Brownian motion: the probability of the event that Brownian motion stays above level l_1 during an unbroken time period of length D_1 occurs before the event that it is below level $-l_2$ during an unbroken time period of length D_2 equals

$$\frac{l_1\sqrt{2/\pi} + \sqrt{D_1}}{(l_1 + l_2)\sqrt{2/\pi} + \sqrt{D_1} + \sqrt{D_2}}.$$

In the special case where $l_1 = l_2 = 0$, we get a formula that is quite similar to the probability found in the classical ruin problem. We will give a proof for this formula using excursion theory, which explains the similarity of both formulas. The same problem will be discussed for symmetric random walk.

Overview of (locally) risk-minimizing hedging strategies and application to unit-linked life insurance contracts

Michèle Vanmaele (speaker) and Nele Vandaele

Follmer and Sondermann [Hedging of Non-Redundant Contingent Claims, in Contributions to Mathematical Economics, Hildenbrand, W. and Mas-Colell, A. (eds.), North-Holland, Elsevier, 1986, 205-223] were the first to describe the riskminimizing hedging strategy for a contingent claim when the underlying risky asset follows a martingale process. Schweizer [A guided tour through quadratic hedging approaches, in Option pricing, interest rates and risk management, E. Jouini, J. Cvitanic, M. Musiela (eds.), Cambridge University Press, 2001, 538-574] extended this theory to describe the locally risk-minimizing hedging strategy for the case that the risky asset follows only a semimartingale process. Up to now in literature, this locally risk-minimizing hedging strategy is not often used or is sometimes used in a wrong way. We will point out the possible pitfalls, explain which conditions are needed for the locally risk-minimizing hedging strategy and how one can determine this strategy. When the risky asset is continuous, we can simply use the risk-minimizing hedging strategy under the minimal martingale measure. However when the risky asset is discontinuous, this equivalence does not longer hold. Further, we will discuss the most important applications and extensions of the locally risk-minimizing hedging strategy. In particular, we will apply the locally risk-minimizing strategy to unit-linked life insurance contracts where the underlying asset is driven by a L/' evy process. When hedging this type of contracts, we are not dealing with a purely financial risk but have to treat a combination of financial and insurance risk.

Wavelet Galerkin schemes for option pricing in multidimensional Lévy models

Christoph Winter

Pricing of multi-asset options using Lévy models leads to partial integro-differential equations in several dimensions. These can be solved with a wavelet Galerkin scheme where sparse tensor product spaces are applied for the discretization to break down the complexity. The resulting matrices are dense since the jump operator is global. Therefore, wavelet compression methods are used to reduce the number of non-zero matrix entries. We focus on algorithmical details of the scheme, in particular on the numerical integration of the matrix coefficients. Since the multidimensional Lévy densities have singularities at the origin and on the axes, variable order composite Gauss quadrature formulas are employed. We give numerical examples.

The explicit solution to a sequential switching problem with non-smooth data

Mihail Zervos

We consider the problem faced by a decision maker who can switch between two random payoff flows. Each of these payoff flows is an additive functional of a general one-dimensional Itô diffusion. There are no bounds on the number or on the frequency of the times at which the decision maker can switch, but each switching incurs a cost, which may depend on the underlying diffusion. The objective of the decision maker is to select a sequence of switching times that maximises the associated expected discounted payoff flow. In this context, we develop and study a model in the presence of assumptions that involve minimal smoothness requirements from the running payoff and switching cost functions, but which guarantee that the optimal strategies have relatively simple forms. In particular, we derive a complete and explicit characterisation of the decision maker's optimal tactics, which can take qualitatively different forms, depending on the problem data.

Initial enlargement of filtrations and entropy of Poisson compensators

Jakub Zwierz

Let μ be a Poisson random measure, $\mathbb{F}$ the smallest filtration satisfying the usual conditions and containing the one generated by μ, and let $\mathbb{G}$ be the initial enlargement of $\mathbb{F}$ with the σ-field generated by a random variable G. We first show that the *mutual information* between the enlarging random variable G and the σ-algebra generated by the Poisson random measure μ is equal to the expected *relative entropy* of the $\mathbb{G}$-compensator relative to the $\mathbb{F}$-compensator of the random measure μ. We then use this link to gain some insight into the changes of Doob-Meyer decompositions of stochastic processes when the filtration is enlarged from $\mathbb{F}$ to $\mathbb{G}$. In particular, we show that if the mutual information between G and the σ-algebra generated by the Poisson random measure μ is finite, then every square integrable $\mathbb{F}$-martingale is a $\mathbb{G}$-semimartingale that belongs to the normed space $\mathcal{S}^1$ relative to $\mathbb{G}$.

Optimal barrier policy for dividend optimization

George Yin (speaker), Qingshuo Song and Hailiang Yang.

Consider an insurance risk model when the dividends are paid to the share holders according to a barrier strategy. A class of algorithms based on stochastic optimization methods is developed to approximate the optimal barrier. In addition to convergence and rates of convergence of the algorithms, numerical results are reported to demonstrate the performance of the algorithms.