SIMION STOILOW INSTITUTE OF MATHEMATICS OF THE ROMANIAN ACADEMY

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Higher order methods: recent advances and open questions

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Abstract: Composite minimization involves a collection of functions which are aggregated in a nonsmooth manner. It covers, in particular cases, nonlinear least-squares, smooth approximation of minimax games, minimization of max-type functions, minimization problems with functional constraints and simple composite minimization problems, where the objective function has a nonsmooth component. We present a higher-order majorization-minimization algorithmic framework for such composite problems (possibly nonconvex). This framework replaces each component in the composite model with a higher-order surrogate such that the corresponding error function has a higher-order Lipschitz continuous derivative. Our algorithmic framework encompasses tensor methods, higher-order proximal methods and even higher-order Gauss-Newton type methods as particular algorithms. We present convergence guarantees (including rates) for these higher-order majorization-minimization algorithms in both convex and non-convex settings. Besides providing a general framework for the design and analysis of composite higher-order methods, in special cases, where complexity bounds are known for some particular (first-order) algorithms, our convergence results recover the existing bounds. Applications to non-linear least-squares (including phase retrieval), control, and functional constrained minimization are presented. Finally, some open questions related to higher-order optimization are discussed.