SIMION STOILOW INSTITUTE OF MATHEMATICS OF THE ROMANIAN ACADEMY

Rank II LGC method: recent progress on several problems in harmonic analysis

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Abstract: Building on the (Rank I) LGC-methodology introduced by the speaker and on the novel perspective employed in the time-frequency discretization of the non-resonant bilinear Hilbert–Carleson operator (joint work with C. Benea, F. Bernicot and M. Vitturi), we develop a new, versatile approach—referred to as Rank II LGC—that has as a consequence the resolution of the following three problems:

- (joint with my postdoc Bingyang Hu) the boundedness of the trilinear Hilbert transform along the moment curve:

$$T_C(f_1, f_2, f_3)(x) := \text{p.v.} \int_{\mathbb{R}} f_1(x-t) f_2(x+t^2) f_3(x+t^3) \frac{dt}{t}, \quad x \in \mathbb{R}\,.$$

- (joint with C. Benea and F. Bernicot) the boundedness of the hybrid trilinear Hilbert transform:

$$T_H(f_1, f_2, f_3)(x) := \text{p.v.} \int_{\mathbb{R}} f_1(x-t) f_2(x+t) f_3(x+t^3) \frac{dt}{t}, \quad x \in \mathbb{R}\,.$$

- (joint with my graduate student Martin Hsu) the boundedness of the curved Carleson–Radon transform:

$$CR(f)(x,y) := \sup_{a \in \mathbb{R}} \left| \text{p.v.} \int_{\mathbb{R}} f(x-t, y-t^2) \, \frac{e^{a\,i\,t^3}}{t} \, dt \right|, \quad (x,y) \in \mathbb{R}^2\,.$$

One of the main difficulties in approaching all of the above problems is the lack of absolute summability for the associated (Rank I) LGC-derived discretized model. In order to overcome this, we design a so-called *correlative* time-frequency model whose control is achieved via the following interdependent elements:

- a sparse-unform decomposition of the input function(s) adapted to an appropriate time-frequency foliation of the phase-space,
- a structural analysis of suitable maximal "joint Fourier coefficients", and
- a level set analysis with respect to the time-frequency correlation set.