KOROVKIN-TYPE THEOREMS FOR A CLASS OF WEAKLY NONLINEAR AND MONOTONE OPERATORS

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Korovkin's theorem (1953) provides a very simple test of convergence to the identity for any sequence $(T_n)_n$ of positive linear operators that map $C([0,1])$ into itself. Precisely, if

$$\lim_{n\to\infty} T_n(f) = f \quad \text{uniformly on } [0,1]$$

for each of the functions 1, x and x^2, then this holds for all functions $f \in C([0,1])$. The same works if $C([0,1])$ is replaced by $C_{2\pi}(\mathbb{R})$ and the triplet of test functions is replaced by 1, cos and sin.

Our talk is aimed to discuss the extension of Korovkin's theorem to a large class of nonlinear operators which are genuine in the context of Choquet's theory of integration. A sample is as follows:

Theorem 1. (See [3], [5]) *Suppose that X is a locally compact subset of the Euclidean space $\mathbb{R}^N$ and E is a vector sublattice of $\mathcal{F}(X)$ that contains the test functions $1,\ \pm\operatorname{pr}_1, \dots,\ \pm\operatorname{pr}_N$ and $\sum_{k=1}^N \operatorname{pr}_k^2$.*

(i) If $(T_n)_n$ is a sequence of monotone and sublinear operators from E into E such that

$$\lim_{n\to\infty} T_n(f) = f \quad \textit{uniformly on the compact subsets of } X$$

for each of the $2N+2$ aforementioned test functions, then this property also holds for all nonnegative functions f in $E \cap C_b(X)$.

(ii) If, in addition, each operator T_n is comonotone additive, then $(T_n(f))_n$ converges to f uniformly on the compact subsets of X, for every $f \in E \cap C_b(X)$.

Notice that in both cases (i) and (ii) the family of testing functions can be reduced to $1,\ -\operatorname{pr}_1, \dots,\ -\operatorname{pr}_N$ and $\sum_{k=1}^N \operatorname{pr}_k^2$ when K is included in the positive cone of $\mathbb{R}^N$. Also, the convergence of $(T_n(f))_n$ to f is uniform on X when $f \in E$ is uniformly continuous and bounded on X.

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