SIMION STOILOW INSTITUTE OF MATHEMATICS OF THE ROMANIAN ACADEMY

IMAR Monthly Lecture

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Overcoming the course of dimensionality: from nonlinear Monte Carlo to the training of neural networks

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Abstract: Partial differential equations (PDEs) are among the most universal tools used in modelling problems in nature and man-made complex systems. Nearly all traditional approximation algorithms for PDEs in the literature suffer from the so-called "curse of dimensionality" in the sense that the number of required computational operations of the approximation algorithm to achieve a given approximation accuracy grows exponentially in the dimension of the considered PDE. With such algorithms it is impossible to approximatively compute solutions of high-dimensional PDEs even when the fastest currently available computers are used. In the case of linear parabolic PDEs and approximations at a fixed space-time point, the curse of dimensionality can be overcome by means of Monte Carlo approximation algorithms and the Feynman-Kac formula. In this talk we present an efficient machine learning algorithm to approximate solutions of high-dimensional PDE and we also prove that deep artificial neural network (ANNs) do indeed overcome the curse of dimensionality in the case of a general class of semilinear parabolic PDEs. Moreover, we specify concrete examples of smooth functions which cannot be approximated by shallow ANNs without the curse of dimensionality, but which can be approximated by deep ANNs without the curse of dimensionality. In the final part of the talk we present some recent mathematical results on the training of neural networks.