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WORKSHOP

„Doctoral Research Days"

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ABSTRACTS:

Alexandra-Ionela Andriciuc: *The PSO algorithm – a particular version.*
***Coordinator**:* **Ionel Popescu** (Univ. Bucharest & IMAR)

ABSTRACT: The PSO algorithm is a very popular one among the practitioners in ML thanks to the elegant coding manner and its interesting parallel with animal behavior. However, the mathematical literature is still lacking a solid background in this regard; for instance, one cannot say much about the convergence of the algorithm, given a precise choice for the hyperparameters. Our purpose is to take advantage of the stochastic intervention from this algorithm - not approximating it with constants as in most previous algebraic works - in order to obtain qualitative results. Hence, classical probabilistic instruments are put to use, aiming to create a context that unifies the rigorous conditions with the go-to values from practice.

Cipriana Anghel: *Heat kernel asymptotics for real powers of Laplacians.*
Coordinator**: **Sergiu Moroianu (IMAR)

ABSTRACT: We prove that some of the heat coefficients in the small-time asymptotic expansion of $e^{−t \Delta^r} are non-local, where $r a (0,1)$, and Δ is a Laplace-type operator over a compact Riemannian manifold. Furthermore, in the special case $r=1/2$, the heat kernel of $\Delta^{1/2}$ is a polyhomogeneous conormal function on the standard blow-up space M_{Heat} of the diagonal at time $t=0$ inside $[0,\infty) \times M \times M$.

Vlad-Raul Constantinescu: *Global Minima of Overparametrized Neural Networks.*
***Coordinator**:* **Ionel Popescu** (Univ. Bucharest & IMAR)

ABSTRACT: We study the geometry of global minima of the loss landscape of overparametrized neural networks. In most optimization problems, the loss function is convex, in which case we only have a global minima, or nonconvex, with a discrete number of global minima. We prove that for a family of activation functions, the locus of global minima of the loss landscape of an overparametrized neural network is a submanifold of R^n. More precisely, if a neural net has n parameters and is trained on d data points, where $n > d$, then the locus M of global minima is an $n-d$ dimensional submanifold of R^n. Also, we give a description of the Hessianevaluated at these global minima.

Hassan Mohsen: *Uniform Estimates for Changing-Sign Transmission Problems: Links to Boundary Triples and Uncertainty Quantification*.
***Coordinator**:* **Victor Nistor** (Univ. Lorraine)

ABSTRACT: Transmission problems are a common occurence in applications, for instance in solid mechanics when an object made of different materials. Another example is provided by the superconductivity phenomenon. The latter example motivates us to study sign-changing transmission problem. An approach for solving such problems is based on the theory of boundary triples. With the help of this theory, we study the existence and the spectrum of the self-adjoint extensions of the operator Pu:=−div(σ¿ u) associated to the transmission problem. This brings us to the question of "uncertainty quantification", which is a topic of great practical importance for engineers, but for which there are few rigorous results. Although this work deals with the three theories above, our main result concerns uncertainty quantification. More precisely, this result consists of estimates that are polynomial in the norms of the coefficients of the equation for the solution of the transmission problem. The crux of this theorem is to provide the piecewise H(k+1) regularity of the solution u of our transmission problem P and the polynomial growth of the corresponding norm of the solution u with respect to the parameters of the system; that is, with respect to the coefficients of the operator P, which are seen as random variables. Consequently, one can integrate the norm of the solution ζ^(−1)Fζ_k+1, as well as the norm of the approximation error with respect to suitable Gaussian measures, a result that is crucial in applications.

Gabriel E. Monsalve: *Atypical values of polynomials and gradient index at infinity.*
***Coordinator**:* **Mihai Tibăr** (Univ. Lille)

ABSTRACT: Let $f : R^2 \to R$ be a polynomial function with isolated singularities. Its index at infinity $ind_\infty(f)$ is defined as the topological degree of the gauss map of f restricted to a big enough circle such that the singular set of f is contained in its interior. We present an effective detection of the atypical values of f based in the detection of the phenomena at infinity that f may have i.e. either vanishing or splitting of fibre components. We also show how this phenomena at infinity influence the index at infinity of f by showing a formula of $ind_\infty(f)$ in terms of the number of vanishing and splitting fibre components.

Alexandru Mustăţea: *Stochastic integration in Riemannian manifolds from a functional-analytic point of view.*
***Coordinator**:* **Radu Purice** (IMAR)

ABSTRACT: We present a construction of the concept of stochastic integration in Riemannian manifolds from a purely functional-analytic point of view. We show that there are infinitely many such integrals, and that any two of them are related by a simple formula. We also find that the Stratonovich and Itô integrals known to probability theorists are two instances of the general concept constructed herein.

Chaima Nefzi: *Doob's ω-transform of parabolic problem for fractional Laplacian.*
***Coordinator**:* **Mounir Bezzarga** (Univ. De Tunis), **Lucian Beznea** (Univ. Bucharest & IMAR)

ABSTRACT: The aim of our work is to give some uniqueness results for nonnegative solutions for a perturbed Dirichlet fractional Laplacian on R^d by using a new version of a Doob's ω−tranform technique. Our method based on the so called (Doob's ω − transformation). The well-known Doob's ω − transform technique is based on the observation that if ω is harmonic on (R^d, dx), then there is a tight connection between the objects relative to (R^d, dx) and those relative to (R^d, $\tilde{dx}$), where R^d is endowed with the density measure $\tilde{dx}$ such that $\tilde{dx} = \omega^2 dx$.

Mihaela-Adriana Nistor: *Risk Measures.*
***Coordinator**:* **Ionel Popescu** (Univ. Bucharest & IMAR)

ABSTRACT: The risk measures are statistical instruments used in predicting the risk and volatility of a portfolio based on historical data. The most popular risk measures are the standard deviation, Sharpe ratio, beta, Value at Risk (VaR), and Conditional Value at Risk (CVaR). What all these have in common is that they are measuring the risk (random variable) with a real number. The downside is that all these classical measures are unable to capture (or predict) the change in the risk regime. Our proposal is a risk measure that is associating to the random variable not a single real value, but a step function. The aim is to capture the magnitude of the risk under different risk regimes. In this presentation, we aim at covering a few theoretical aspects of the proposed risk measure.