

Seminarul “Nicolae Popescu” Number Theory

*Analytic aspects in the evaluation of multiple zeta values  
and multiple Hurwitz zeta values*

Cezar Lupu

(University of Pittsburgh, USA)

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**Abstract.** In this talk, we shall discuss about some new results in the evaluation of some multiple zeta values (MZV). After a careful introduction of the multiple zeta values (Euler-Zagier sums) we point out some conjectures back in the early days of MZV and their combinatorial aspects. At the core of our talk, we focus on Zagier’s formula for the multiple zeta values,  $\zeta(2, 2, \dots, 2, 3, 2, 2, \dots, 2)$  and its connections to Brown’s proofs of the conjecture on the Hoffman basis and the zig-zag conjecture of Broadhurst in quantum field theory. Zagier’s formula is a remarkable example of both strength and the limits of the motivic formalism used by Brown in proving Hoffman’s conjecture where the motivic argument does not give us a precise value for the special multiple zeta values  $\zeta(2, 2, \dots, 2, 3, 2, 2, \dots, 2)$  as rational linear combinations of products  $\zeta(m)\pi^{2n}$  with  $m$  odd. The formula is proven indirectly by computing the generating functions of both sides in closed form and then showing that both are entire functions of exponential growth and that they agree at sufficiently many points to force their equality. By using the Taylor series of integer powers of arcsin function and a related result about expressing rational zeta series involving  $\zeta(2n)$  as a finite sum of  $\mathbb{Q}$ -linear combinations of odd zeta values and powers of  $\pi$ , we derive a new and direct proof of Zagier’s formula in the special case  $\zeta(2, 2, \dots, 2, 3)$ . Towards the end of the talk, we present a more general formula for  $\zeta(2, 2, \dots, 2, m)$ , with  $m \geq 3$  integer. We also discuss similar results for the multiple t-values.