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Instantaneous shrinking of supports and single point extinction for viscous Hamilton-Jacobi equations with fast diffusion

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Abstract: For a large class of non-negative initial data, the solutions to the quasilinear viscous Hamilton-Jacobi equation $\partial_t u - \Delta_p u + |\nabla u|^q = 0$ in $(0, \infty) \times \mathbb{R}^N$ are known to vanish identically after a finite time when $2N/(N+1) < p \leq 2$ and $q \in (0, p-1)$. Further properties of this extinction phenomenon are established in this talk: *instantaneous shrinking* of the support is shown to take place if the initial condition u_0 decays sufficiently rapidly as $|x| \to \infty$, that is, for each $t > 0$, the positivity set of $u(t)$ is a bounded subset of $\mathbb{R}^N$ even if $u_0 > 0$ in $\mathbb{R}^N$. This decay condition on u_0 is also shown to be optimal by proving that the positivity set of any solution emanating from a positive initial condition decaying at a slower rate as $|x| \to \infty$ is $\mathbb{R}^N$ for all times prior to the extinction time. The time evolution of the positivity set is also studied: on the one hand, it is included in a fixed ball for all times if it is initially bounded (*localization*). On the other hand, it converges to a single point at the extinction time for a class of radially symmetric initial data, a phenomenon referred to as *single point extinction*. This behavior is in sharp contrast with what happens when q ranges in $[p-1, p/2)$ and $p \in (2N/(N+1), 2]$ for which we show *complete extinction*. We stress that instantaneous shrinking and single point extinction take place in particular for the semilinear viscous Hamilton-Jacobi equation when $p = 2$ and $q \in (0, 1)$ and these results are new even in this case.

Work in collaboration with **Philippe Laurençot (Inst. de Mathématiques de Toulouse, France)** and **Christian Stinner (Felix-Klein-Zentrum, TU Kaiserslautern, Germany)**.