Weyl-Pedersen calculus on coadjoint orbits of nilpotent Lie groups.

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The Weyl-Pedersen calculus we are dealing with is the remarkable correspondence $a \mapsto \mathrm{Op}^{\pi}(a)$ constructed by N.V. Pedersen in [Matrix coefficients and a Weyl correspondence for nilpotent Lie groups. *Invent. Math.* **118** (1994), no. 1, 1–36] as a generalization of the pseudo-differential Weyl calculus on $\mathbb{R}^n$. Here $\pi\colon G \to \mathbb{B}(\mathcal{H})$ is any unitary irreducible representation of a connected, simply connected, nilpotent Lie group G, the symbol a can be any tempered distribution on the coadjoint orbit $\mathcal{O}$ corresponding to π by the orbit method and $\mathrm{Op}^{\pi}(a)$ is a linear operator in the representation space $\mathcal{H}$, which is in general unbounded.

We present here boundedness and compactness properties for the operators obtained by the Weyl-Pedersen calculus in the case of the irreducible unitary representations of nilpotent Lie groups that are associated with flat coadjoint orbits. We use spaces of smooth symbols satisfying appropriate growth conditions expressed in terms of invariant differential operators on the coadjoint orbit under consideration. Our method also provides conditions for these operators to belong to one of the Schatten ideals of compact operators. In the special case of the Schrödinger representation of the Heisenberg group we recover some classical properties of the pseudo-differential Weyl calculus, as the Calderón-Vaillancourt theorem, and the Beals characterization in terms of commutators.