Scientific Report

Regarding implementation of the project
Research in Operator Algebra and Applications to Number Theory
during the period 1 September 2013-30 September 2016

During 1 September 2013-30 September 2016, the research in Operator Algebra and Applications to Number Theory was continued. In this context were studied Hecke algebras and their representations, convex structures in the space of representations of sofic groups, crossed products of pro-C^*-algebras, Hopf $H(n)$ algebras, their representations and the associated cyclic cohomology groups, automorphic forms and δ-geometries. There were also studied the structure of dimension groups and their relation to ergodic theory and number theory and the commuting squares associated to finite groups and finite dimensional Hopf algebras. The outcome of this research are the papers in the attached reference list.

The research directions and their outcome are as follows:

• Florin Rădulescu analyzed the deformation algebra of the hyperbolic upper halfplane, obtained from the Berezin quantization, by using the discrete series of representations of $PSL(2,\mathbb{R})$. He determined the structure of the associated unbounded Hochschild cocycle, which is a possible invariant for the associated von Neumann algebras of type II. The outcome of this study is the paper [27], a component of the theme of this project (point 1, Problematics and Objectives from the description of the research project).

In this direction the author has analyzed the structure of matrix coefficients associated with the representations and the correspondence with matrix coefficients of the representations of the Hecke algebra associated to the inclusion $PSL(2,\mathbb{Z}) \subseteq PSL(2,\mathbb{Q})$.

Analyzing the symmetries of these matrix coefficients one obtains a representation of the adelic completion of the larger group. The importance of this line of research is the relation to the Ramanujan-Petersson conjecture on the spectrum of the Hecke operators. It is intended to find an estimate on the "gap"predicted by the conjecture. The goal of the research is to understand from the point of view of operator algebras the existence of such a "gap" in the spectrum of the Hecke operators.

Florin Rădulescu has continued in the year 2014 the analysis of the representations of the Hecke algebras and the study of continuity properties of these representations. This investigation is an important part of the research in the thematics of the project (point 1, Problematics and Objectives from the description of the research project).

The objects that are studied are obtained from representations π of pairs

$\Gamma \subseteq G$, where G is a discrete group, Γ an almost normal subgroup (that is all defect indices $[\Gamma : \Gamma \cap \sigma\Gamma\sigma^{-1}], \sigma \in G$ are finite). The classical case corresponds to the inclusion of the modular group $PSL(2,\mathbb{Z}) \subseteq PGL(2,\mathbb{Q})$. One obtains unitary (projective) representations of the group G with the property that $\pi|_\Gamma$ is a multiple of the left regular representation of the group Γ. The last mentioned property of the representation π gives, through a construction similar to the construction of Gelfand triples of vector spaces, a lattice of vector spaces. These spaces consist of vectors that are invariant for the action of groups Γ as above. On these spaces, the action π of the group G extends to representation, denoted by $\overline{\pi}$, of a profinite extension $\overline{G}$ of G. For Γ and G as above, one obtains the p-adic groups $PSL(2,\mathcal{Q}_p)$. It is proven that in the case of the representations from the discrete series of the group $PSL(2,\mathbb{R})$, the above construction leads to spaces of automorphic forms associated to the modular group, the scalar product being the classical Petersson scalar product.

The classical problem of the Ramanujan-Petersson estimates is ultimately reformulated in this context as problem of non-commutative harmonic analysis: one asks if the representation $\overline{\pi}$ is weakly contained in the left regular representation of the profinite extension $\overline{G}$ of the group G. The above procedure realizes a bijective correspondence between the representations π and $\overline{\pi}$.

Within this framework one reformulates the Ramanujan-Petersson conjecture for Maass forms. Surprisingly, this corresponds to the problem of lifting of a representation into the outer automorphism group of the II_1 factor associated to the action of the modular group. This research is developed in the paper [29], revised in final form in December 2014. The paper is submitted for publication.

In the following, an important part of the research will be dedicated to the determination of the conditions that guarantee that the character formula found above determines the location of the character of the representation $\overline{\pi}$ in the space of the representations weakly contained in the regular representation. This is a test for the validity of the Ramanujan-Petersson conjectures.

The paper [25], contains a theory, dual to the one introduced above. This is used to formalize in operator algebra terms the action of Hecke operators on Maass form. In this work one uses the cost theory of an equivalence relation, introduced by D. Gaboriau. Using this method one proves that the equivalence relation obtained by reduction of the equivalence relation defined by the action of the group $\mathrm{PGL}(2, \mathbb{Z}[\frac{1}{p}])$ on the upper, complex half plane, to the fundamental domain of the action of the modular group, is a probability, measure preserving countable equivalence relation that is treeable (in the sense of Gaboriau) of cost $\frac{p+1}{2}$ ($p \geq 3$ is a prime number). In particular

the algebra generated by the Hecke operators may be identified with radial algebra associated to the action of a free group with $\frac{p+1}{2}$-generators, acting on the upper halfplane. This result was one of the motivations for the paper [22].

The paper [24] contains an analysis of the matrix coefficient representation of the Hecke algebra in the case of unitary representations π from the discrete series π, that are unitarily equivalent, when restricted to Γ, to the left regular representation of Γ . The representations of the Hecke algebra are obtained from the analysis of the matrix coefficients of the representation π. This results of this paper are used in an essential way in the paper [23].

The paper [26] contains an exposition (based on notes by N. Ozawa) of a part of the results contained in a former paper by F. Rădulescu (arXiv:0802.3548). This later paper was one of the motivation for the part 1 of the objectives of the project.

In the year 2015, Florin Rădulescu continued the study of properties of the Calkin algebra and of the representations into the Calkin algebra by left and right convolution operators. This study is extended a result of Akeman Ostrand on the structure of the convolution algebra to the case of the action of the canonical groupoid associated to the pair consisting of a discrete group and an almost normal subgroup. This result is used to obtain asymptotical estimates for the Hecke operators and is part of the problematics/objectives 1 of the project. The results are contained in the paper ([28], submitted for publication).

In the paper ([23]), part of the same thematics of the project, Rădulescu has obtained a correspondence between properties of unitary representations, in the discrete series of a semisimple Lie group of rank 1 and the corresponding unitary representations of the profinite completion of a dense subgroup.

In 2016, F. Rădulescu, (within the framework of the problematic/objectives 1 of the project) has continued the study of the correspondence between representations in the discrete series of semisimple Lie group of rank 1 and the unitary representation of the profinite completion of a dense subgroup. The paper [23] was completed, following also the referee remarks and it was published in the journal Russian Mathematical Surveys in 2016. It has been proven that there exists an explicit formula for the character of the unitary representation of the profinite completion. As an example it has been observed that this corresponds to a concrete calculation of the character of the unitary representation of $\mathrm{PGL}(2, Q_p)$ that defines the action of the Hecke operators on automorphic forms. This allows to comparison of this character with the characters of the irreducible representations in the principal series of $\mathrm{PGL}(2, Q_p)$.

The results in this paper give an explicit correspondence between the two

types of representations and allow the transfer of the "spectral gap" problem of Hecke operators to an operator algebra problem. This correspondence is the starting point of a joint research collaboration with Alexandru Popa (IMAR). By analogy with the Plancherel formula, one expects to obtain the character of the unitary representation as a explicit convex combination (or a limit of thereof) of characters in the principal series of $\mathrm{PGL}(2, Q_p)$. This would allow an harmonic analysis argument for P. Deligne estimates on the eigenvalues for the Hecke operators acting on automorphic forms.

• Liviu Păunescu developed a study of the space of sofic representations of a group. Using the existing definitions, this space is non-separable, the main cause for which this space is difficult to analyze. The article "All automorphisms of the universal sofic group are class-preserving" [21], published in Revue Roumaine de Mathématiques Pures et Appliquées, is investigating a possible new definition of this space (part of Point 4 of Problems/Objective of the project description). Also the study of the restriction map on the space of sofic representations was initiated, an important part of this project.

Liviu Păunescu continued in 2014 the study of convex structures on the space of sofic representations of a group G, that in the following lines we denote by $SofG$. This is an essential part of Point 4 of Problems/Objective of the project description. In the article "Convex structures revisited" [20], accepted to Ergodic Theory and Dynamical Systems, a large part of this objective has been accomplished. The article contains a complete description of the extremal points of the convex structure on $SofG$. It remains to be seen if extremal points are enough to generate the whole space $SofG$, i.e. a Krein-Milman type result. A positive answer would reduce the study of sofic representations to the study of extreme representations.

In the same article the author manages to construct two groups H and G such that the restriction map form $SofG$ to $SofH$ is not surjective. This is a first result showing an obstruction for soficity in general. We plan to generalize this construction about non-surjectivity of the restriction map. The proof is based on the notion of "expanders", that is about a graph with good connectivity properties. This notion has been intensively studied, both in mathematics and informatics, as it has multiple applications including in computer networks. The article presents a first application of this notion of "expanders" to metric approximation properties.

We have to understand if this phenomena of non-surjectivity of the restriction map can be used to construct a non-sofic group. The existence of two groups G_1 and G_2 such that the images of the restriction maps to $SofH$ do not intersect would solve this problem. Indeed, the free product of G_1 and G_2 amalgamated over H would be non-sofic.

In the paper "Almost commuting permutations are near commuting per-

mutations" ([21]), Goulnara Arzhantseva and Liviu Păunescu proved a stability result of the commutator relator with respect to the Hamming distance. The article is submitted for publication. To the best of author's knowledge, our result is the first one for infinite groups.

In the article "A Generalisation to Birkhoff - von Neumann Theorem" Florin Radulescu and Liviu Paunescu studied Birkhoff - von Neumann theorem in context II_1. This research is a joint work part of points 1 and 4 of the "Problematics and Objectives" section from the description of the research project.

This classical result says that permutation matrices are extreme points in the set of double-stochastic matrices. To understand the II_1 context, we must consider (X, μ) a standard probability space and $E \subset X^2$ an equivalence relation that is measure preserving. Matrices are now considered over E and automorphisms of (X, μ) play the role of permutation matrices (for more details see section 1 of the cited article in this paragraph). In this context the general theorem is false, as we have shown in Section 2.2 of this article. By developing some combinatorics techniques, we still managed to save a large part of this result, see Section 3, Theorem 3.9. In section 6 we discuss an important application of this theorem for Hecke operators.

In 2016, Liviu Paunescu studied connections between sofic and hyperlinear groups, as described in Point 4 of "C1 Problems" of the project description. He obtains conditions under which a unitary in an ultraproduct of matrices is actually an ultraproduct of permutations. This is the subject of a paper in preparation.

• Maria Joiţa continued the study of the crossed product of pro-C^*-algebras, a component of the theme of this project (point 1, Problematics and Objectives from the description of the research project). Crossed product of a pro-C^*-algebra A by an inverse limit action α of a locally compact group G was defined by N.C. Philips using the Arens-Michael decomposition of the pro-C^*-algebra A and of the action α. In the paper [9], published in "Annals Functional Analysis", it is defined the crossed product of a pro-C^*-algebra A by an action α of a locally compact group G as a universal pro-C^*-algebra with respect to non-degenerate covariant representations. It is proved that such an object exists for a strong bounded action. Also, it is proved that the maximal (minimal) tensor product of the crossed product (reduced crossed product) of a pro-C^*-algebra A by a strong bounded action α and a pro-C^*-algebra B is isomorphic with the crossed product (reduced crossed product) of the maximal (minimal) tensor product of A and B by action $\alpha \otimes \mathrm{id}$. Unfortunately, if the action α is strongly bounded, then there is another family of C^*-seminorms on A which is equivalent to the family of C^*-seminorms which defines the topology on A, and α is an inverse limit

action with respect to this family of C^*-seminorms.

The paper [10], published in "Banach Journal of Mathematical Analysis" is devoted the study of Morita equivalence for Hilbert modules over C^*-algebras. Hilbert C^*-modules X and Y over C^*-algebras A and B are Morita equivalent in the sense of M. Skeide if there is an $A-B$ imprimitivity bimodule M such that $X \otimes M = Y$. Two full Hilbert C^*-modules X and Y over A and B are Morita equivalent in the sense of M. Skeide if and only if the C^*-algebras of compact operators associated to their are isomorphic. Hilbert C^*-modules X and Y over C^*-algebras A and B are Morita equivalent in the sense of M. Joita and M.S. Moslehian if the C^*-algebras of compact operators associated to their are Morita equivalent. In case of full countably generated Hilbert C^*-modules over σ -unital C^*-algebras this definition coincides with Skeide's definition of stable Morita equivalence. In this paper, it is introduced a direct and constructive notion of Morita equivalence for Hilbert C^*-modules based on the notion of imprimitivity bimodules by analogy with the notion of Morita equivalence for C^*-algebras. Actions of the two C^*-algebras on the imprimitivity bimodule are replaced here by actions of the two Hilbert C^*-modules. The main advantage of our definition is that we give a explicit list of axioms for the imprimitivity bimodule (like the original case of C^*-algebras). However there are certain drawbacks: we could show that our notion is an equivalence relation only for Hilbert C^*-modules with unit vectors. In the case of full Hilbert C^*-modules with unit vectors, we show that the three notions of Morita equivalence coincide.

In the paper [13], it is examined under which conditions we can associate a pro-C^*-algebra to a pro-C^*-correspondence. In case (X, A, φ) is a pro-C^*-correspondence with the property that the ideals $\ker p_\lambda, \lambda \in \Lambda$ ($\{p_\lambda\}_{\lambda\in\Lambda}$ is the family of C^*-seminorms which defines the topology on A) are invariant, a universal pro-C^*-algebra can be associated to it. This construction generalizes the construction of crossed products by Hilbert pro-C^*-bimodules and the construction of pro-C^*-crossed products by strong bounded automorphisms.

In 2015 Maria Joiţa continued the study of the crossed product of pro-C^*-algebras, a component of the theme of this project (point 1, Problematics and Objectives from the description of the research project). The notion of crossed product of a pro-C^*-algebra A by a Hilbert pro-C^*-bimodule (X, A) was introduced by M. Joiţa and I. Zarakas [Crossed products by Hilbert pro-C^*-bimodules, Studia Math. 215(2013), 2,139–156] and it is a generalization of the notion of crossed product of a pro-C^*-algebra A by an inverse limit automorphism α. If α is an inverse limit automorphism of a pro-C^*-algebra A, then $X_\alpha = \{\xi_x; x \in A\}$ is a Hilbert $A-A$ pro-C^*-bimodule with the bimodule structure defined as $\xi_x a = \xi_{xa}$, respectively $a\xi_x = \xi_{\alpha^{-1}(a)x}$, and the inner products are defined as $\langle \xi_x, \xi_y \rangle_A = x^* y$,

respectively ${}_A\langle \xi_x, \xi_y\rangle = \alpha(xy^*)$, and the crossed product $A \times_\alpha \mathbb{Z}$ of A by α is isomorphic with the crossed product of A by the Hilbert pro-C^*-bimodule (X_α, A). It is known that given two (pro-) C^*-algebras A and B and an inverse limit automorphism α of A, then the (pro-) C^*-algebras $(A \otimes_{\min} B) \times_{\alpha\otimes_{\min}\mathrm{id}} \mathbb{Z}$ and $(A \times_\alpha \mathbb{Z}) \otimes_{\min} B$ are isomorphic as well as the (pro-) C^*-algebras $(A \otimes_{\max} B) \times_{\alpha\otimes_{\max}\mathrm{id}} \mathbb{Z}$ and $(A \times_\alpha \mathbb{Z}) \otimes_{\max} B$. It is natural to ask if these results can be extended in the context of crossed products by Hilbert C^*-bimodules. In the paper [11], the author shows that given a Hilbert (pro-)C^*-bimodule (X, A) and a (pro-)C^*-algebra B, the crossed product $(A \otimes_{\min} B) \times_{X\otimes_{\min}B} \mathbb{Z}$ associated to $(X \otimes_{\min} B, A \otimes_{\min} B)$ is isomorphic to the minimal tensor product $(A \times_X \mathbb{Z}) \otimes_{\min} B$ of the crossed product associated to (X, A) and B. Also, she proves that if (X, A) is a full Hilbert (pro-)C^*-bimodule and B is a (pro-)C^*-algebra, then the crossed product $(A \otimes_{\max} B) \times_{X\otimes_{\max}B} \mathbb{Z}$ associated to $(X \otimes_{\max} B, A \otimes_{\max} B)$ is isomorphic to the maximal tensor product $(A \times_X \mathbb{Z}) \otimes_{\max} B$ of the crossed product associated to (X, A) and B. As an application, it is proved that the crossed product $A \times_X \mathbb{Z}$ of a pro-C^*-algebra A by a full Hilbert pro-C^*-bimodule (X, A) is a nuclear pro-C^*-algebra if and only if A is nuclear. Another proof of this result, in the case of C^*-algebras, was given by Katsura [On C^*-algebras associated with C^*-correspondences, J. Funct. Anal. 217(2004) 366–401].

In the paper [12], M. Joiţa, R-B. Munteanu and I. Zarakas showed that given a Hilbert pro-C^*-bimodule (X, A), the vector space $L_A(A, X)$ of all adjointable module morphisms from A to X has a natural structure of Hilbert $M(A)-M(A)$ pro-C^*-bimodule. It is denoted by $(M(X), M(A))$ and is called the multiplier pro-C^*-bimodule associated to (X, A). The authors introduce the notion of the strict topology on $(M(X), M(A))$ and prove that $M(X)$ is complete with respect to the strict topology and X can be identified with a sub-bimodule of $M(X)$ which is dense in $M(X)$ with respect to the strict topology. Also, they investigate the relationship between the crossed product $A \times_X \mathbb{Z}$ of a pro-C^*-algebra A by a Hilbert pro-C^*-bimodule X over A, the crossed product $M(A) \times_{M(X)} \mathbb{Z}$ of the multiplier algebra $M(A)$ of A by the multiplier bimodule $M(X)$ of X and the multiplier algebra $M(A \times_X \mathbb{Z})$ of $A \times_X \mathbb{Z}$. It is showed that the crossed product associated to a full Hilbert pro-C^*-bimodule (X, A) can be identified with a pro-C^*-subalgebra of the crossed product associated to $(M(X), M(A))$ and the crossed product associated to $(M(X), M(A))$ can be identified with a pro-C^*-subalgebra of the multiplier algebra of the crossed product associated to (X, A). As an application, the authors prove that given an inverse limit automorphism α of a nonunital pro-C^*-algebra A, the crossed product of $M(A)$ by $\overline{\alpha}$, the extension of α to $M(A)$, can be identified with a pro-C^*-subalgebra of the multiplier algebra $M(A \times_\alpha \mathbb{Z})$ of $A \times_\alpha \mathbb{Z}$.

In 2016, Maria Joiţa continued the study of the Cuntz-Pimsner algebra associated to a pro-C^*-correspondence. If (X, A, φ_X) and (Y, B, φ_Y) are two pro-C^*-correspondences such that the ideals $\ker p_\lambda, \lambda \in \Lambda$ and $\ker q_\delta, \delta \in \Delta$ ($\{p_\lambda\}_{\lambda\in\Lambda}$ and $\{q_\delta\}_{\delta\in\Delta}$ are the families of C^*-seminorms which define the topologies on A, respectively B) are positively invariant, then $(X \otimes Y, A \otimes_{\min} B, \varphi_{X\otimes Y})$, where $\varphi_{X\otimes Y} = j \circ (\varphi_X \otimes \varphi_Y)$, j is the canonical pro-C^*-morphism from $L(X)\otimes_{\min} L(Y)$ to $L(X\otimes Y)$, is a pro-C^*-correspondence, called the tensor product of the pro-C^*-correspondences (X, A, φ_X) and (Y, B, φ_Y), with the property that the ideals $\ker \nu_{(\lambda,\delta)}, (\lambda, \delta) \in \Lambda \times \Delta$ are positively invariant, where $\{\nu_{(\lambda,\delta)}\}_{(\lambda,\delta)\in\Lambda\times\Delta}$ is the family of C^*-seminorms which defines the topology on $A \otimes_{\min} B$. Then, according to a result from [13], $(X \otimes Y, A \otimes_{\min} B, \varphi_{X\otimes Y})$ admits a Cuntz-Pimsner algebra. In the paper [14], submitted for publication, M. Joiţa describes the Cuntz-Pimnsner algebra associated to the tensor product of two pro-C^*-correspondences in terms of the gauge actions. A Hilbert pro-C^*-bimodule (X, A) can be regarded as a pro-C^*-correspondence and the crossed product $A\times_X Z$ of A by X is isomorphic with O_X, the Cuntz-Pimsner algebra associated to this pro-C^*-correspondence, [13]. In [11, Theorem 3.4], it is proved that if (X, A) is a Hilbert pro-C^*-bimodule and B is a pro-C^*-algebra, then the pro-C^*-algebras $(A \times_X \mathbb{Z})\otimes_{\min} B$ and $(A \otimes_{\min} B)\times_{X\otimes B} Z$ are isomorphic. The author extends this result for Cuntz-Pimsner algebras associated to pro-C^*-correspondences. She proves that if (X, A, φ_X) is a pro-C^*-correspondence and B is a pro-C^*-algebra such that $\mathcal{J}_{X\otimes B} = \mathcal{J}_X \otimes_{\min} B$, then the Cuntz-Pimsner algebra $\mathcal{O}_{X\otimes B}$ associated to the pro-C^*-correspondence $(X\otimes B, A\otimes_{\min} B, \varphi_{X\otimes B})$ is isomorphic with the minimal tensor product $\mathcal{O}_X \otimes_{\min} B$ of the pro-C^*-algebras $\mathcal{O}_X$ and B. As in case of C^*-correspondences (see, [A. Morgan,Cuntz-Pimsner algebras associated to tensor product of C^*-correspondences, arXiv:1510.04959v1 (2015)]), she shows that if (X, A, φ_X) and (Y, B, φ_Y) are two ideal-compatible pro-C^*-correspondences, Katsura nondegenerate, then $\mathcal{O}_{X\otimes Y}$ is isomorphic with the fixed point algebra of the action $z \mapsto \alpha_z \otimes \beta_{z^{-1}}$ of the unit tor $\mathbb{T}$ on $\mathcal{O}_X \otimes_{\min} \mathcal{O}_Y$, where α and β are the gauge actions associated to universal covariant representations (π_X, t_X) , respectively (π_Y, t_Y).

The notion of a scattered C^*-algebra was introduced by H. E. Jensen (1977) as a noncommutative generalization of the notion of a scattered Hausdorff topological space. The paper [15] (presented at 7th European Congress of Mathematics 7ECM in Berlin, July 18 - 22, 2016), submitted for publication, concerns scattered pro-C^*-algebras and crossed product of scattered pro-C^*-algebras. The author introduces the notion of a scattered pro-C^*-algebra as a generalization of the notion of a scattered C^*-algebra and she gives conditions for a pro-C^*-algebra to be scattered. It is showed that a pro-C^*-algebra A is scattered if and only if all the factors $A_\lambda, \lambda \in \Lambda$ form

its Arens-Michael decomposition are scattered C^*-algebras. Also, she proves that the crossed product $A \times_\alpha G$ of a scattered pro-C^*-algebra A by a "perfect" inverse limit action α of a commutative compact group G is a scattered pro-C^*-algebra if and only if the fixed point algebra is a scattered pro-C^*-algebra.

• Remus Nicoară has continued the investigation of commuting squares C of the form $C(G)$ and $C(A)$, where G is a finite group, or more generally a finite dimensional Hopf algebra A. Two definitions are introduced: $d(C) =$ the unfazed defect of C, $d'(C) =$ the fazed defect. This is a generalization of existing notions of defect for Fourier matrices. One computes the maximum number of directions in which C may be deformed in the class of all commuting squares, respectively in the class of all non-isomorphic commuting squares. It is possible to construct $d(C)$ analytic families. Applications of these results are investigated in the theory of Hopf algebras. This study is a part of the thematics of the project (point 3, Problematics and Objectives from the description of the research project). This result, stated as one of the objectives of the project, and presented at the 25-th Operator Theory Conference, Timi?oara, 2014, has been published in Rev. Roumaine de Math ematiques Pures et Appliques, Vol 59(2014), no 2, pp 245-254, ([18]).

During 2015, Nicoara investigated the moduli space of complex Hadamard matrices, with applications to quantum information theory. Two types of problems were tackled:

1. The structure of the moduli space of $n \times n$ Hadamard matrices around the Fourier matrix F_n. Nicoara constructed new multi-dimensional analytic families of Hadamard matrices containing the matrix F_n. These lead to constructions of mutually unbiased bases (MUB's), which have applications in quantum computing.

2. Finiteness results for certain classes of Hadamard matrices. Nicoara showed that there exist only finitely many Hadamard matrices of circulant core in dimensions $p + 1$ with p prime. The result is based on a version of the uncertainty principle for the group Z_p, due to Terry Tao. More work in this direction also lead Nicoara to the discovery of new permutation-type Hadamard matrices.

In 2016, Remus Nicoară, in [19] (coauthor J. White), submitted for publication to the J. Functional Analysis, constructs new parametric families of commuting squares, obtained as analytic deformations of group commuting squares associated to finite groups G. In the case when G is cyclic this yields new parametric families of complex Hadamard matrices, containing the Fourier matrix.

• Henri Moscovici has investigated in the paper ([16]) the Hopf algebras $H(n)$ and the associated cyclic cohomology. These have been investigated by

de A. Connes and the author in a paper dedicated to transversal elliptic operators in which they determine their characteristic classes. The same authors have proved that the Hopf algebra Hopf $H(1)$ and their characteristic classes appear in the theory of modular forms, in connection with Hecke operators. More precisely, the cocycles representing the cyclic cohomology classes of the Hopf algebra $H(1)$ have interesting arithmetical interpretation. In the paper [16], are obtained concrete formulae for the cocycles representing the classes of relative cyclic cohomology of the Hopf algebra $H(n)$ modulo $GL(n)$, for every positive integer n. These are formulae are similar to those representing equivariant Chern classes in Geometry and Topology. This is a very interesting problem since the transfer of this formulae in the cohomological theory of Hopf algebras, brings to the formulae a larger potential of applicability, as it was the case for the algebra $H(1)$. This study is part of the thematics of the project (point 5, Problematics and Obj., description of the research project).

In 2015, Henri Moscovici, in the paper ([17], in collaboration with Bahram Rangipour accepted for publication in 2015 in Journal of Noncommutative Geometry, related to problematics/objectives 5 of the project) constructs a variant K_n of the Hopf algebra H_n, which acts directly on the noncommutative model for the space of leaves of a general foliation rather than on its frame bundle. It is proven that the Hopf cyclic cohomology of K_n is isomorphic to that of the pair $(H_n; gl_n)$ and thus consists of the universal Hopf cyclic Chern classes. These classes are realized in terms of geometric cocycles.

• Florin Boca, during August-October 2014, started the study, in collaboration with J. Tseng, of a problem that connects the structure of dimension groups with ergodic theory and number theory. In spite of the (sometimes spectacular) progress achieved in the classification of C*-algebras, the structure of dimension groups of AF-algebras, sub-summed by the so-called "unimodular conjecture", remains far from being elucidated, and appears to be an important area where results and techniques from number theory, especially from Diophantine approximation, might be applied. The proposed approach employs some modern techniques from the ergodic theory of homogeneous flows, instead of classical continued fraction algorithms. Boca and Tseng have already obtained partial results concerning the structure of certain dimension groups of small rank. Instead of publishing these partial results, our plan is to focus on the general problem, which is certainly more difficult but could be approached with these techniques.

During 2015 Florin Boca continued the research regarding problematics/objectives 2 of the project. Certain types of dynamical systems, such as Bernoulli or Markov shifts, have natural noncommutative analogues in operator algebras and have been intensively studied. An important class of

non-invertible transformations (endomorphisms) of the interval [0,1] consists of transformations associated to continued fraction expansions. The best known are the classical Gauss shift and the Farey shift, but they do not have an obvious noncommutative counterpart. A suitable noncommutative analogue of a number $\theta \in [0,1]$ is provided by the Effros-Shen algebra, whose dimension group captures its regular continue fraction expansion $CF(\theta)$. The corresponding noncommutative analogue of the interval [0,1], capturing both topological and number theoretical features, is provided by a certain AF algebra A, introduced by Mundici and recently studied by Boca and Eckhardt, whose primitive ideals contains a family of ideals I_θ such that $A/I_\theta \cong C_\theta$. Boca has studied the problem of extending the Perron-Frobenius type operators associated to these two transformations to unital completely positive maps on the AF-algebra A, acting on its primitive ideal space in a compatible way with the Gauss and Farey shifts. A first extension of this type was constructed for the Gauss shift by Eckhardt in 2011. Our work in preparation investigates this type of construction and also tries to introduce such an extension for the Farey shift.

In 2016, Boca has continued working on a research theme related to problematics/objectives 2. He considered the sieve inequality, known to play a key role in a series of important problems in the analytic theory of numbers. This consists in estimating, as precise as possible, the largest eigenvalue $\lambda_1(A)$ of a certain $N \times N$ semi-positive definite matrix A^*A, naturally associated to a quadratic form that involves roots of unity. Our approach is from a noncommutative perspective, analyzing more generally the distribution of all eigenvalues. For arithmetic applications the most interesting situation is $N \asymp Q^2$ with $Q \to \infty$. After performing thorough numerical experiments, Olivier Ramaré conjectured in 2007 the existence of the limiting distribution of the eigenvalues of matrices A^*A, scaled by $\frac{1}{N}$, when $N \sim \alpha Q^2$, with $Q \to \infty$ and $\alpha > 0$ constant. The classical sieve inequality guarantees the compactness of the support of the limiting probability measure on $[0, \infty)$, if this limit exists. The current work ([2], joint with Maksym Radziwill) solves this conjecture, by establishing convergence of the eigenvalues of the matrix A^*A (scaled by $\frac{1}{N}$) as $Q \to \infty$. Previously, this convergence was know only for the second order moments, result due to Ramaré. Furthermore, we were able to obtain an explicit (yet complicated) description of the moments of the limiting distribution, and also to show their continuity in α. The main ingredients in the proof include the large sieve inequality, results concerning correlations of Farey fractions (obtained in a joint 2005 paper with Zaharescu), basic harmonic and functional analysis techniques, as well as a series of elementary (yet delicate) number theoretical estimates. At the time being we were not able to obtained a more precise description of the limiting

distribution, but we plan to continue to investigate this difficult problem in the future.

• Alexandru Buium (collaborator in the project). In the paper "Differential modular forms attached to new forms mod p" ([3]), are extended older results of the author regarding automorphic forms. In previous papers, the author has associated to forms ("new" automorphic forms), differential modular forms of order 2. These are an essential "block of representations" and they play an important role in the theory of automorphic forms. These are dual forms. The goal of the paper is to introduce a higher order version, in which it is proven that to modular forms of weight $k \in \{3, ..., p\}$ one associates differential modular forms of order 2 and weight $k' \in \{1, 2, ..., p-2\}$. This work is related to the thematics of the project, related to δ-geometries and desingularization of quotient spaces (point 1, Problematics and Objectives from the description of the research project).

Furthermore, in 2015, in relation to the above problematics an in relation to Objective 5, Buium has considered in the papers ([4],[5],[7],[6],[8]), statements on the spectrum of the integers that are similar to some key concepts from Riemannian geometry such as: metrics, Chern connections, curvature, etc. Theorems are stated to the effect that the spectrum of the integers has a non-vanishing curvature.

Conferences

• Within the project, together with the Italian-French research group in Noncommutative Geometry GDRE GREFI-GENCO and Romanian-French laboratory LEA Math Mode, during September 1-8 2013 in Poiana Brasov, the Noncommutative Geometry and Applications Workshop was organized. (http://imar.ro/NGA/description.php).

From the Romanian side organizer was Florin Rădulescu. The Workshop addressed young researchers working in this research area, and was composed of a series of lectures by senior researchers and presentations by junior researchers. Apart from the Romanian-France-Italian group, Profesor N. Ozawa (Univ. Tokyo) took part in the event with a series of four expositions, as well as two researchers from Portugal. From the team of this research project, Florin Boca had a series of four expositions, while Liviu Păunescu and Florin Rădulescu, each had a plenary talk.

• Liviu Păunescu gave a talk (December 2013) at University Sapienza, Roma, within the Sofic Groups Seminar (led by Prof. Roberto Conti), about his recent work on the convex structure of the space of embeddings inside the universal sofic group.

• The following members of the project research team took part at the "25th International Conference on Operator Theory", Timişoara, 30 June-5 July, 2014: L. Păunescu (title of the talk "Sofic representations that cannot

be extended"), M. Joiţa (title of the talk "Crossed products by Hilbert pro-C^*-bimodules versus tensor products"), F. Rădulescu (plenary talk, title "On the theory of spaces of vectors invariant to the action of a discrete group"), H. Moscovici (plenary talk, title "Morita equivalence and noncommutative curvature"), R. Nicoară (title of the talk "The defect of a group-type commuting square"). The following experts in the field of Operator Algebras participated at this conference (invited from this research project) with plenary talks strictly on the subject: S. Vaes, (Univ. Leuven, title "Classification of crossed product von Neumann algebras"), A. Thom (Univ. Leipzig, title "Random walks and invariant random forests"), M. Junge (Univ Chicago, Urbana Champaign, title "Actions of q-gaussian algebras"), V. Capraro (Univ S. Hampton, title "Groups associated to II_1 factors"), H. Li (University of Buffalo, NY, title "Homoclinic group, IE group, and expansive algebraic actions").

• Alexandru Buium, Univ. New Mexico has participated at the activity of the project in the period 30 June -31 July 2014, as invited professor. The theme of the research was the interplay between different methods of desingularization of quotient spaces: crossed products and δ-geometries (part of thematics/ point 1 of the objectives). The outcome of this research is the paper [3]. Alexandru Buium gave a talk in the monthly series of the Institute of Mathematics of the Romanian Academy "Simion Stoilow" with the title "Arithmetic analogue of some concepts in Riemannian geometry", on 9 July 2014.

• A workshop "Geometric Group Theory" was organized at the Institute of Mathematics of the Romanian Academy, 30 September -4 October, 2014, in collaboration with the bilateral program România-Austria "Geometry and Analysis of Linear Soficity " (program directed from the Romanian part by L. Păunescu and from the Austrian part by G. Arzhantseva). On behalf of this research project, have been invited at the Institute of Mathematics of the Romanian Academy "S. Stoilow", with plenary expositions, strictly related to the thematics of the project: T. Ceccherini (Univ. Benevento, Italia), M. Cavaleri (Univ. Roma 2, Italia), Ioana Claudia Lazăr (Univ. Vest Timişoara). Organizers were G. Arzhantseva (Univ Viena, Austria), F. Rădulescu and L. Păunescu. L. Păunescu and F. Rădulescu gave plenary expositions. Goulnara Arzhantseva, Univ Viena, Austria, gave in the monthly series of conferences of the Institute of Mathematics of the Romanian Academy "Simion Stoilow" a conference with the title "Approximations of infinite groups results and challenges"on 1 Oct. 2014.

• Liviu Păunescu si F. Rădulescu gave each a 1 hour lecture, in the Seminar of Geometric Group Theory, in relation to the bilateral collaboration program Romania -Austria, GALS (April 2014).

• F. Rădulescu gave a conference at the University of Geneve, on the structure of Hecke algebras in April 2014.

• A mini workshop on Operator Algebras was organized at the Institute of Mathematics of the Romanian Academy in August 2014 with plenary expositions by members of the team:: Liviu Păunescu, Florin Boca, Maria Joiţa, Florin Rădulescu and by Radu Purice (IMAR).

• F. Rădulescu gave a plenary conference at the annual Session of the Romania Mathematical Society, Romania, with the title "Hilbert 17-th Problem and Noncommutative Geometry ", Iasi, 25 Oct., 2014.

• Supported by IDEI programme and by the bilateral cooperation grant Romania-Austria, GALS (2014-2015), L. Păunescu was in Vienna for a research visit on two occasions: from March 1 to May 15 and from October 25 to November 7 at University of Vienna.

Liviu Păunescu gave a 2 hours talk at the seminar "Geometric Group Theory", title "On the Birkhoff - von Neumann theorem in type II1 setting".

• At the Congress of Romanian Mathematicians, Iaşi, June 2015, Florin Boca, Maria Joita, Liviu Păunescu and Florin Rădulescu participated from the project side (the latter as an organiser of the Operator Algebras section). Florin Boca gave a talk titled "The distribution of rational numbers and ergodic theory". Maria Joiţa gave a talk titled "Pro-C*-correspondences". Liviu Păunescu gave a talk titled "Almost commuting permutations are near commuting permutations".

• Florin Rădulescu was a plenary speaker at the 34th Workshop on Geometric Methods in Physics, Bialowieza, Polonia (organised by University of Bialystok) where he gave a conference on "Berezin deformation Quantization and Number Theory", during June 6-11 2015.

• We organised a mini-workshop of Operator Algebras at the Institute of Mathematics of the Romanian Academy in July 13-14 2015. The plenary speakers were: Liviu Păunescu, Florin Boca, Maria Joiţa, Florin Rădulescu, Henri Moscovici.

• A workshop "Group Theory and Operator Algebra" was organized at the Institute of Mathematics of the Romanian Academy, 29 September -3 October, 2015, in collaboration with the bilateral program România-Austria "Geometry and Analysis of Linear Soficity " (program directed from the Romanian part by L. Păunescu and from the Austrian part by G. Arzhantseva). On behalf of this research project, have been invited at the Institute of Mathematics of the Romanian Academy "S. Stoilow", with plenary expositions, strictly related to the thematics of the project: T. Ceccherini (Univ. Benevento, Italia), M. Cavaleri (Univ. Roma 2, Italia), Paul Schupp (University of Chicago, Champaign Urbana) and Florin Boca (presently at the University of Chicago Champaign Urbana). Organizers were G. Arzhant-

seva (Univ Viena, Austria), F. Rădulescu and L. Păunescu. L. Păunescu, M. Joiţa and F. Rădulescu gave plenary expositions. Paul Schupp, University of Chicago Champaign Urbana, gave, in collaboration with workshop, in the monthly series of conferences of the Institute of Mathematics of the Romanian Academy "Simion Stoilow" a conference with the title "Why group theory and computability are inextricably intertwined" on 1 Oct. 2015.

• In 2016, on the occasion of the semester "Measurable Group Theory", at the Schroedinger Institute, Vienna, in partnership with the University of Vienna, Liviu Pănescu (in the period January March 2016) and Florin Rădulescu (in the period 1-22 February 2016) have participated to the semester activities, in connection with the thematics of the project (problematics/objectives 1,2), continuing the collaboration with the research group "Geometry and Analysis on Groups" (University of Viena). Liviu Păunescu presented a conference at the University of Vienna with the title "On the Birkhoff - von Neumann theorem in type II_1 setting".

• In the "26-th International Conference on Operator Theory", Timişoara, 26 June-1 July, 2016 have participated the following members of the team of the project: L. Păunescu (title of the presentation "Birkhoff - von Neumann theorem in type II_1 setting"), F. Rădulescu (plenary, title of the presentation "Spectral gaps and Hecke operators"), H. Moscovici (plenary, title of the presentation "Morita equivalence and noncommutative curvature").

At the same conference, between 26 June - 3 July, for scientific collaboration within the objectives of the project, the following professors have participated: Hari Bercovici (Indiana University, Bloomington, SUA, plenary, title of the presentation "Outliers for random matrix models"), Victor Kaftal (University of Cincinnati, SUA, plenary, title of the presentation "The integer in Kadison's Pythagorean Theorem and essential codimension"), Jean Renault (University of Orleans, France plenary, title of the presentation "Random walks on Bratteli diagrams").

• In accordance with the objectives of the project (Section 1) during 3-18 July 2016, F. Rădulescu has participated in Portugal, at the Conference "Operator Algebra and Operator Theory" at the Institute Tecnico Superior, Lisabona, with a plenary conference entitled "Quantum Dynamics and Operator Algebras with Applications In Number Theory". Also F. Rădulescu gave a talk, with the same title at the University of Aveiro in the Fuctional Analysis Seminar. During its stay Florin Rădulescu has collaborated (within the thematics of the objectives 1 in the project) with the researcher Paolo Pinto, at the Instituto Tecnico Superior , Lisbon, on some problems regarding the strucuture of pro C^*-algebras, identifying a new possible approach regarding the problematics of the objectives 1 and 5 in the project.

• Within the thematics of the project (objectives 1 and 5), during 18-22 July 2016, Maria Joiţa has participated, in Berlin at the 7-th European Congress of Mathematics) where she gave a talk with the title "Scattered locally C^*-algebras".

• F. Rădulescu has made a scientific collaboration visit within the thematics of the project (objectives 1 and 4) at the Hausdorff Research Institute of Mathematics, Berlin in the Semester Von Neumann Algebras, at the invitation of the professor Vaughan F. R. Jones. on this occasion , F. Rădulescu gave a talk entitled "Operator algebras and number theory".

• L. Păunescu has made a scientific collaboration visit within the thematics of the project (objectives 4) at the Mathematics Institute Jussieu, University Paris 7, at the invitation of Professor G. Skandalis. L. Păunescu gave a talk Seminar, in the Operator Algebra entitled "Birkhoff - von Neumann theorem in type II_1 setting".

• Within the thematics of the project (objectives 1, 4 and 5), during 26-30 September 2016, a workshop entitled "Sofic groups and Operator Algebra" was held at the Institute of Mathematics of the Romanian Academy, Bucharest. At this workshop have participated during 24 - 30 September 2016, for a scientific collaboration, within the thematics of the project (objectives 1,4,5) the professors T. Ceccherini (Universita del Sannio, Benevento, Italy), G. Arzhantseva (Univ Viena, Austria) and the doctoral student Oren Becker (Hebrew University). Prof. T. Ceccherini gave a plenary lecture in the series of Monthly lectures of the Institute of Mathematics of the Romanian Academy,in collaboration with the workshop,entitled "The Garden of Eden Theorem: old and new". G. Arzhantseva (Univ Viena, Austria), O. Becker, L. Păunescu, Maria Joiţa, F. Rădulescu gave plenary lectures in the workshop.

Presentations in seminaries

Starting with Oct. 2013, we initiated at the Institute of Mathematics of the Romanian Academy, two scientific seminars, dedicated to the subject of the project. One of the seminaries is directed by Liviu Păunescu and Florin Rădulescu. In this seminar were presented basic elements on the structure of sofic groups. In the second seminary, directed by Florin Rădulescu, on Operator Algebras it was presented, by Liviu Păunescu in 2013: the Connes-Feldmann-Weiss Theorem on the structure of amenable equivalence relations and in 2014: theory of cost of a measurable equivalence relation (Gaboriau).

In the Operator Theory and Operator Algebra Seminar at the Institute of Mathematics of the Romanian Academy, Remus Nicoară held an exposition entitled "Groups, commuting squares and Hadamard matrices" (May 2016),

Liviu Păunescu a held an exposition entitled "The Theorem of Birkoff - von Neumann in the II_1 context " (May 2016) si F. Rădulescu held three expositions entitled "Endomorphisms of spaces of virtual vectors fixed by a discrete group" (Martie-Mai 2016).

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