Partial differential inclusions in reflexive Orlicz-Sobolev spaces

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ABSTRACT. We study the weak solvability of PDI's of the type

$$-\text{div}(a(|\nabla u|)\nabla u) \in \partial_C f(x, u(x)), \text{ in } \Omega,$$

subject to Dirichlet boundary condition in a domain $\Omega \subset \mathbb{R}^N$ with Lipschitz boundary $\partial\Omega$. Here, $a : (0, \infty) \to (0, \infty)$ is such that

$$\Phi(t) = \int_0^t a(s)s\, ds,$$

defines an N-function and the corresponding Orlicz-Sobolev space $W_0^1 L^\Phi(\Omega)$ is reflexive. The function $f : \Omega \times \mathbb{R} \to \mathbb{R}$ is locally Lipschitz w.r.t. the second variable and ∂_C denotes the Clarke subdifferential of $t \mapsto f(x, t)$.

Using a minimization technique and the Zero Altitude Mountain Pass Theorem for locally Lipschitz functionals the existence of at least one weak solution is established. A multiplicity alternative is also proved via nonsmooth Schechter theory. More precisely, we show that either the problem possesses at least two nontrivial weak solutions or a rich family of negative eigenvalues.

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