Finite time extinction diffusive Hamilton-Jacobi equations

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The behavior near the extinction time is identified for non-negative solutions to the diffusive Hamilton-Jacobi equation with critical gradient absorption

$$\partial_t u - \Delta_p u + |\nabla u|^{p-1} = 0 \quad \text{in } (0,\infty) \times \mathbb{R}^N ,$$

and fast diffusion $2N/(N+1) < p < 2$. Given a non-negative and radially symmetric initial condition with a non-increasing profile which decay sufficiently fast as $|x| \to \infty$, we show that the corresponding solution u to the above equation approaches a *uniquely determined separate variable solution* of the form

$$U(t,x) = (T_e - t)^{1/(2-p)} f_*(|x|), \quad (t,x) \in (0,T_e) \times \mathbb{R}^N ,$$

as $t \to T_e$, where T_e denotes the finite extinction time of u and f_* is the solution to an ODE. A cornerstone of the convergence proof is an underlying *variational structure* of the equation, which can be seen as a gradient flow of a weighted functional. Also, the selected profile f_* is the unique non-negative solution to a second-order ordinary differential equation which decays exponentially at infinity. One cornerstone difficulty when proving the uniqueness of a special profile f_* among the infinitely many existing steady states is that **no monotonicity argument** seems available and it is overcome by the construction of an appropriate *Pohozaev functional*. We strongly believe that this new technique is a starting point for approaching a number of open problems about special profiles where monotonicity arguments are not available. We will also report briefly on results for the subcritical extinction case

$$\partial_t u - \Delta_p u + |\nabla u|^q = 0 \quad \text{in } (0,\infty) \times \mathbb{R}^N ,$$

where $0 < q < p-1$, where the extinction mechanism departs strongly from the previous case and new phenomena appear.

Results obtained in collaboration with **Philippe Laurençot** (IMT, Toulouse, France) and **Christian Stinner** (UT Kaiserslautern, Germany).