HOMEWORK 2

1) Determine the points of intersection of C_1 and C_2 together with their intersection numbers: a) $C_1 = V(x_0x_2^2 - x_1^3)$, $C_2 = V(x_0x_2^2 - x_1^3 - x_0x_1^2)$ b) $C_1 = V(x_0x_2^2 - x_1^3 - x_0x_1^2)$, $C_2 = V(x_1^2 + x_2^2 + x_0x_1)$, $C_3 = V(x_0^2 + 2x_1^2 + x_2^2 + 3x_0x_1)$.

2) Determine the Puiseux characteristic of : a) $(t^2, t^3 + t^4)$ b) $(t^2, t^4 + t^6 + t^7)$
c) $y = x^{5/2} + x^{13/4}$ d) $y = x^{7/6} + x^{37/24} + x^{20/9}$.

3) Draw pictures illustrating the succesive stages of blowing up for obtaining good resolutions of: (i) $y^4 = x^5$, (ii) $y^3 = yx^3$.

4) Let B be a branch with Puiseux characteristic $(2; 5)$. List the sequence of multiplicities $m_i(B)$ occuring in a minimal good resolution of B. Determine the proximity relations arising and write down the proximity matrix P. Draw the dual resolution graph $\Gamma^+(B)$. Sketch the pattern of strict transforms of B and the exceptional curves at each stage of the resolution.

5) Let $f = x^5 + x^2y^2 + y^5$. Show, by blowing up, that $C = \{f = 0\}$ has two branches at the origin O, each with Puiseux characteristic $(2; 3)$ and with distinct tangents.

6) Let $C_1 = \{(y^3 - x^4)^2 = x^7y^2\}$, $C_2 = \{(y^2 - x^3)^3 = x^{11}\}$. By repeated blowing up, and noting the sequence of multiplicities obtained, show that each C_i has a single branch, determine their Puiseux characteristics and resolution graphs. By checking the two sequences of blowings up, determine how many infinitely near points C_1 and C_2 have in common. Hence obtain the resolution graph for $C_1 \cup C_2$. Determine the intersection number $C_1 \cdot C_2$ at the origin.

7) Classify singularities of multiplicity 3 up to equisingularity by listing all possibilities for Puiseux characteristics and mutual intersection numbers of the branches.

8) Classify irreducible singularities of multiplicity 4 up to equisingularity by listing all possibilities for Puiseux characteristics. In each case, determine the sequence of multiplicities.

9) Show that the singular points given by $y^3 + x^6 = 0$ and $y^3 + yx^4 = 0$ are equisingular but are not related by a holomorphic change of coordinates. (Hint: Expand a general change of coordinates as a power series, substitute into one curve and equate succesively coefficients of terms of low degree).

10) Suppose the Newton polygon N_f is a single edge from $(a, 0)$ to $(0, b)$, with a, b coprime. Show that the sequence of multiplicities for $B = \{f = 0\}$ arises from applying the Euclidean algorithm (to determine the greatest common divisor) to a and b. Deduce that B is equisingular to $x^a = y^b$.