HOMEWORK 1

1) Show that (t^3, t^2+t^4) is a good parametrization of a curve B and find an equation for B.
2) For $y^3 - 9x^3y - x^4 = 0$ find a parametrization $x = t^3$, $y = \psi(t)$ and obtain the first four non-vanishing coefficients of ψ.
3) Show that $f(x, y) = y^3 + xy^2 + x^4$ is reducible in $\mathbb{C}\{x, y\}$. Find the first two terms of the Puiseux series for each branch of $f(x, y) = 0$.
4) Find the Newton polygons of the branches of $x^4 + x^3y + y^5 = 0$.
5) How many branches at the origin have the following curves:
a) $x^2 + 2xy + 2y^2 = 0$, b) $x^6 - x^2y^3 - y^5 = 0$, c) $x - y + (x + y)^2 = 0$?
6) Calculate the intersection number $B_1 \cdot B_2$ for $B_1 : x = t^4$, $y = t^6 + t^7$ and $B_2 : x = t^6$, $y = t^9 + t^{10}$.
7) Find singular points and tangent lines there for:
a) $y^3 - y^2 + x^3 - x^2 + 3x^2y + 3xy^2 + 2xy = 0$, b) $x^4 - x^2y^2 + y^4 = 0$,
c) $x^3 + y^3 - 3x^2 - 3y^2 + 3xy + 1 = 0$, d) $y^2 + (x^2 - 5)(4x^4 - 20x^2 + 25) = 0$.
8) Find the singular points of the projective quartics:
a) $(x_0x_2 - x_1^2)^2 = x_1x_2^3$, b) $(x_0x_2 - x_1^2)^2 = x_1^3x_2$.
9) If a curve C has degree d and a point p of multiplicity $m_p(C) = d$ show that C is a union of lines through p.
10) Let p be point on a curve $C = \{f(x, y) = 0\}$. Show that p is a node if and only if $(f_{xy}^2 - f_{xx}f_{yy})(p) \neq 0$.