HOMEWORK 1

1) Show that \((t^3, t^2+t^4)\) is a good parametrization of a curve \(B\) and find an equation for \(B\).

2) For \(y^3 - 9x^3y - x^4 = 0\) find a parametrization \(x = t^3, y = \psi(t)\) and obtain the first four non-vanishing coefficients of \(\psi\).

3) Show that \(f(x, y) = y^3 + xy^2 + x^4\) is reducible in \(\mathbb{C}\{x, y\}\). Find the first two terms of the Puiseux series for each branch of \(f(x, y) = 0\).

4) Find the Newton polygons of the branches of \(x^4 + x^3y + y^5 = 0\).

5) How many branches at the origin have the following curves:
   a) \(x^2 + 2xy + 2y^2 = 0\), b) \(x^6 - x^2y^3 - y^5 = 0\), c) \(x - y + (x + y)^2 = 0\)?

6) Calculate the intersection number \(B_1 \cdot B_2\) for \(B_1 : x = t^4, y = t^6 + t^7\) and \(B_2 : x = t^6, y = t^9 + t^{10}\).

7) Find singular points and tangent lines there for:
   a) \(y^3 - y^2 + x^3 - x^2 + 3x^2y + 3xy^2 + 2xy = 0\), b) \(x^4 - x^2y^2 + y^4 = 0\),
      c) \(x^3 + y^3 - 3x^2 - 3y^2 + 3xy + 1 = 0\), d) \(y^2 + (x^2 - 5)(4x^4 - 20x^2 + 25) = 0\).

8) Find the singular points of the projective quartics:
   a) \((x_0x_2 - x_1^2)^2 = x_1x_2^3\), b) \((x_0x_2 - x_1^2)^2 = x_1^3x_2\).

9) If a curve \(C\) has degree \(d\) and a point \(p\) of multiplicity \(m_p(C) = d\) show that \(C\) is a union of lines through \(p\).

10) Let \(p\) be point on a curve \(C = \{f(x, y) = 0\}\). Show that \(p\) is a node if and only if \((f_{xy}^2 - f_{xx}f_{yy})(p) \neq 0\).