Preface

This is a book about doing model theory without an underlying logical system. It teaches us how to live without concrete models, sentences, satisfaction and so on. Our approach is based upon the theory of institutions, which has witnessed a vigorous and systematic development over the past two decades and which provides an ideal framework for true abstract model theory. The concept of institution formalizes the intuitive notion of logical system into a mathematical object. Thus our model theory without underlying logical systems and based upon institution theory may be called 'institution-independent model theory'.

Institution-independent model theory has several advantages. One is its generality, since it can be easily applied to a multitude of logical systems, conventional or less conventional, many of the latter kind getting a proper model theory for the first time through this approach. This is important especially in the context of the recent high proliferation of logics in computing science, especially in the area of formal specification. Then there is the advantage of illuminating the model theoretic phenomena and its subtle network of causality relationships, thus leading to a deeper understanding which produces new fundamental insights and results even in well worked traditional areas of model theory.

In this way we study well established topics in model theory but also some newly emerged important topics. The former category includes methods (in fact much of model theory can be regarded as a collection of sometimes overlapping methods) such as (elementary) diagrams, ultraproducts, saturated models and studies about preservation, axiomatizability, interpolation, definability, and possible worlds semantics. The latter category includes methods of doing model theory 'by translation', and Grothendieck institutions, which is a recent successful model theoretic framework for multi-logic heterogeneous environments. The last two chapters (14 and 15) digress from the main topic of the book in that they present some applications of institution-independent model theory to specification and programming and Chap. 13 shows how to integrate proof theoretic concepts to institution-independent model theory (including a general approach to completeness).

This book is far from being a complete encyclopedia of institution-independent model theory. While several important concepts and results have not been treated here, we believe they can be approached successfully with institutions in the style promoted by our work. Most of all, this book shows *how* to do things rather than provides an exhaustive account of all model theory that can be done institution-independently. It can be used by any working user of model theory but also as a resource for learning model theory.

From the philosophical viewpoint, the institution-independent approach to model theory is based upon a non-essentialist, groundless, perspective on logic and model theory, directly influenced by the doctrine of *śunyata* of the Madhyamaka Prasangika school within Mahayana Buddhism. The interested reader may find more about this connection in the essay [54]. This has been developed mainly at Nalanda monastic university about 2000 years ago by Arya Nāgārjuna and its successors and has been continued to our days by all traditions of Tibetan Buddhism. The relationship between Madhyamaka Prasangika thinking and various branches of modern science is surveyed in [176].

I am grateful to a number of people who supported in various ways the project of institution-independent model theory in general and the writing of this book in particular. I was extremely fortunate to be first the student and later a close friend and collaborator of late Professor Joseph Goguen who together with Rod Burstall introduced institutions. He strongly influenced this work in many ways and at many levels, from philosophical to technical aspects, and was one of the greatest promoters of the non-essentialist approach to science. Andrzej Tarlecki was the true pioneer of doing model theory in an abstract institutional setting. Till Mossakowski made a lot of useful comments on several preliminary drafts of this book and supported this activity in many other ways too. Grigore Roşu and Marc Aiguier made valuable contributions to this area. Lutz Schröder made several comments and gave some useful suggestions. Achim Blumensath read very carefully a preliminary draft of this book and helped to correct a series of errors. I am indebted to Hans-Jürgen Hoenhke for encouragement and managerial support. Special thanks go to the former students of the Informatics Department of "Şcoala Normală Superioară" of Bucharest, namely Marius Petria, Daniel Găină, Andrei Popescu, Mihai Codescu, Traian Şerbănuţă and Cristian Cucu. They started as patient students of institution-independent model theory only to become important contributors to this area. Finally, Jean-Yves Béziau greatly supported the publication and dissemination of this book. I acknowledge financial support for writing this book from the CNCSIS grants GR202/2006 and GR54/2007.

Ploieşti,

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Contents

Chapter 1

Introduction

Model theory is in essence the mathematical study of semantics, or meaning, of logic systems. As it has a multitude of applications to various areas of classical mathematics, and of logic, but also to many areas of informatics and computing science, there are various perspectives on model theory which differ slightly. A rather classical viewpoint is formulated in [32]:

Model theory = logic + universal algebra.

A rather different and more radical perspective which reflects the success of model theoretic methods in some areas of classical mathematics is given in [99]:

Model theory = algebraic geometry - fields.

From a formal specification viewpoint, in a similar tone, one may say that

Model theory = logical semantics - specification.

Each such viewpoint implies a specific way in developing the key concepts and the main model theory methods; it also puts different emphasis on results. For example while forcing is a very important method for the applications of model theory to conventional logic, it plays a very little role in computing science. On the other hand, formal specification theory requires a much more abstract view on model theory than the conventional one. The institution theory of Goguen and Burstall [30, 75] arose out of this necessity.

Institutions. The theory of institutions is a categorical abstract model theory which formalizes the intuitive notion of a logical system, including syntax, semantics, and the satisfaction relation between them. Institutions constitute a model-oriented meta-theory on logics similarly to how the theory of rings and modules constitute a meta-theory for classical linear algebra. Another analogy can be made with universal algebra versus particular algebraic structures such as groups, rings, modules, etc., or with mathematical analysis over Banach spaces versus real analysis.

The notion of institution was introduced by Goguen and Burstall in the late 1970s [30] (with the seminal journal paper [75] being printed rather late) in response to the population explosion of specification logics with the original intention of providing a proper abstract framework for specification of, and reasoning about, software systems. Since then institutions have become a major tool in development of the theory of specification, mainly because they provide a language-independent framework applicable to a wide variety of particular specification logics. It became standard in the field to have a logic system captured as the institution underlying a particular language or system, such that all language/system constructs and features can be rigorously explained as mathematical entities and to separate all aspects that depend on the details of the particular logic system from those that are general and independent of this logic system by basing the latter on an arbitrary institution. All well-designed specification formalisms follow this path, including for example CASL [10] and CafeOBJ [57].

Recently institutions have also been applied to computing science fields other than formal specification; these include ontologies and cognitive semantics [73], concurrency [138], and quantum computing [31].

Institution-independent model theory. This means the development of model theory in the very abstract setting of arbitrary institutions, free of any commitment to a particular logic system. In this way we gain another level of abstraction and generality and a deeper understanding of model theoretic phenomena, not hindered by the largely irrelevant details of a particular logic system, but guided by structurally clean causality. The latter aspect is based upon the fact that concepts come naturally as presumed features that "a logic" might exhibit or not and are defined at the most appropriate level of abstraction; hypotheses are kept as general as possible and introduced on a by-need basis, and thus results and proofs are modular and easy to track down regardless of their depth. Access to highly non-trivial results is also considerably facilitated, which is contrary to the impression of some people that such general abstract approaches produce results that are trivial. As Béziau explains in [20]:

> "This impression is generally due to the fact that these people have a concrete-oriented mind, and that something which is not specified [n.a. concretely] has no meaning for them, and therefore universal logic [n.a. institution-independent model theory in our case] appears as a logical abstract nonsense. They are like someone who understands perfectly what is Felix, his cat, but for whom the concept of cat is a meaningless abstraction. This psychological limitation is in fact a strong defect because, ... [n.a. as this book shows], what is trivial is generally the specific part, not the universal one [n.a. the institution-independent one] which requires what is the fundamental capacity of human thought: abstraction."

The continuous interplay between the specific and the general in institution-independent model theory brings a large array of new results for particular non-conventional logics, unifies several known results, produces new results in well-studied conventional areas, reveals previously unknown causality relations, and dismantles some which are usually assumed as natural.

Institution-independent model theory also provides a clear and efficient framework for doing logic and model theory ‘by translation (or borrowing)’ via a general theory of mappings (homomorphisms) between institutions. For example, a certain property P which holds in an institution I' can be also established in another institution I provided that we can define a mapping $I \to I'$ which ‘respects’ P.

Institution-independent model theory can be regarded as a form of ‘universal model theory’, part of the so-called ‘universal logic’, a recent trend in logic promoted by Bèziau and others [21].

Other abstract model theories. Only two major abstract approaches to logic have a model theoretic nature and are therefore comparable to the institution-independent model theory.

The so-called “abstract model theory” developed by Barwise and others [12, 13] however keeps a strong commitment to conventional concrete systems of logic by explicitly extending them and retaining many of their features, hence one may call this framework “half-abstract model theory”. In this context even the remarkable Lindström characterization of first order logic by some of its properties should be rather considered as a first order logic result rather than as a true abstract model theoretic one.

Another framework is given by the so-called “categorical model theory” best represented by the works on sketches [63, 88, 181] or on satisfaction as cone injectivity [5, 6, 7, 120, 118, 116]. The former just develops another language for expressing (possibly infinitary) first order logic realities. While the latter considers models as objects of abstract categories, it lacks the multi-signature aspect of institutions given by the signature morphism and the model reducts, which leads to severe methodological limitations. Moreover in these categorical model theory frameworks, the satisfaction of sentences by the models is usually defined rather than being axiomatized.

By contrast to the two abstract model theoretic approaches mentioned above, institutions capture directly the essence of logic systems by axiomatizing the satisfaction relationship between models and sentences without any initial commitment to a particular logic system and by emphasizing propertly the multi-signature aspect of logics.

Book content. The book consists of four parts.

In the first part we introduce the basic institution theory including the concept of institution and institution morphisms, and several model theoretic fundamental concepts such as model amalgamation, (elementary) diagrams, inclusion systems, and free models. We develop an ‘internal logic’ for abstract institutions, which includes a semantic treatment to Boolean connectives, quantifiers, atomic sentences, substitutions, and elementary homomorphisms, all of them in an institution-independent setting.

The second part is the core of our institution-independent model theoretic study because it develops the main model theory methods and results in an institution-independent setting.

The first method considered in this part is that of ultraproducts. Based upon the well-established concept of categorical filtered products, we develop an ultraproduct fundamental theorem in an institution-independent setting and explore some of its immediate consequences, such as ultrapower embeddings and compactness.

The chapter on saturated models starts by developing sufficient conditions for directed co-limits of homomorphisms to retain the elementarity. This rather general version of Tarski's elementary chain theorem is a prerequisite for a general result about existence of saturated models, later used for developing other important results. We also develop the complementary result on uniqueness of saturated models. Here the necessary concept of cardinality of a model is handled categorically with the help of elementary extensions, a concept given by the method of diagrams. We develop an important application for the uniqueness of saturated models, namely a generalized version of the remarkable Keisler-Shelah result in first order model theory, "two models are elementarily equivalent if and only if they have isomorphic ultrapowers".

A good application of the existence result for saturated models is seen in the preservation results, such as "a theory has a set of universal axioms if and only if its class of models is closed under 'sub-models"'. We develop a generic preservation-by-saturation theorem. Such preservation results might lead us straight to their axiomatizability versions. One way is to assume the Keisler-Shelah property for the institution and to use a direct consequence of the fundamental ultraproducts theorem which may concisely read as "a class of models is elementary if and only if it is closed under elementary equivalence and ultraproducts".

Another method to reach an important class of axiomatizability results is by expressing the satisfaction of Horn sentences as categorical injectivity. This leads to general quasi-variety theorems such as "a class of models is closed under products and 'sub-models' if and only if it is axiomatizable by a set of (universal) Horn sentences" and variety theorems such as "a class of models is closed under products and 'sub-models' and 'homomorphic images' if and only if it is axiomatizable by a set of (universal) 'atoms"'.

All axiomatizability results presented here are collected under the abstract concept of 'Birkhoff institution'.

The next topic is interpolation. The institution-independent approach brings several significant upgrades to the conventional formulation. We develop here three main methods for obtaining the interpolation property, the first two having rather complementary application domains. The first one is based upon a semantic approach to interpolation and exploits the Birkhoff-style axiomatizability properties of the institution (captured by the above mentioned concept of Birkhoff institution), while the second, inspired by the conventional methods of first order logic, is via Robinson consistency. The third one is a borrowing method across institutions.

We next treat definability, again with rather two complementary methods, via Birkhoff-style axiomatizability and via interpolation. While the latter represents a generalization of Beth's theorem of conventional first order model theory, the former reveals a causality relationship between axiomatizability and definability.

The final chapter of the second part of the book is devoted to possible worlds (Kripke) semantics and to extensions of the satisfaction relation of abstract institutions to modal satisfaction. By applying the general ultraproducts method to possible worlds semantics, we develop the preservation of modal satisfaction by ultraproducts together with its semantic compactness consequence.

The third part of the book is devoted to special modern topics in institution theory, such as Grothendieck constructions on systems of institutions with applications to heterogeneous multi-logic frameworks, and an extension of institutions with proof theoretic concepts. For the Grothendieck institutions we develop a systematic study of lifting of important properties such as theory co-limits, model amalgamation, and interpolation, from the level of the 'local' institutions to the 'global' Grothendieck institution. We present a rather striking application of the interpolation result for Grothendieck institutions, which leads for example to a quite surprising interpolation property in the Horn fragment of conventional first order logic. The chapter on proof theory for institutions introduces the concept of proof in a simple way that suits the model theory, explores proof theoretic versions of compactness and internal logic, and presents general soundness results for institutions with proofs. The final part of this chapter develops a general sound and complete Birkhoff-style proof system with applications significantly wider than that of the Horn institutions.

The last part presents a few of the multitude of applications of institution-independent model theory to computing science, especially in the areas of formal specification and logic programming. This includes structured specifications over arbitrary institutions, the lifting of a complete calculus from the base institution to structured specifications, Herbrand theorems and modularization for logic programming, and semantics of logic programming with pre-defined types.

The concepts introduced and the results obtained are systematically illustrated in the main text by their applications to the model theory of conventional logic (which includes first order logic but also fragments and extensions of it). There are only two reasons for doing this. The first is to build a bridge between our approach and the conventional model theory culture. The second reason has to do with keeping the material within reasonable size. Otherwise, while conventional (first-order) model theory has been historically the framework for the development of the main concepts and methods of model theory, one of the main messages of this book is that these do not depend on that framework. Any other concrete logic or model theory could be used as a benchmark example in this book, and in fact we do this systematically in the exercise sections with several less conventional logics.

How to use this book. The material of this book can be used in various ways by various audiences both from logic and computing science. Students and researchers of logic can use material of the first two parts (up to Chap. 11 included) as an institution-independent introduction to model theory. Working logicians and model theorists will find in this monograph a novel view and a new methodological approach to model theory. Computer scientists may use the material of the first part as an introduction to institution theory, and material from the third and the fourth parts for an advanced approach to topics from the semantics of formal specification and logic programming. Also, institution-independent model theory constitutes a powerful tool for workers in formal specification to perform a systematic model theoretic analysis of the logic underlying the particular system they employ.

Each section comes with a number of exercises. While some of them are meant to help the reader accommodate the concepts introduced, others contain quite important results and applications. In fact, in order to keep the book within a reasonable size, much of the knowledge had to be exiled to the exercise sections.

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