Three decades of institution theory

Răzvan Diaconescu

1. Introduction

30 years have passed since the introduction by Joseph Goguen and Rod Burstall of the concept of 'institution' (in [14] under the name 'language'). Since then institution theory has gradually developed from a simple and strikingly elegant general category theoretic formulation of the informal notion of logical system into an important trend of what is now called 'universal logic', with substantial applications and implications in both logic and computing science.

During this period of time many things happend. Very sadly, Joseph Goguen passed away in 2006, leaving behind an incredibly diverse scientific inheritance that will benefit many generations to come. It will perhaps take another several decades to fully understand the implications of his ideas. Rod Burstall has retired from the academia years ago. New young scientists from various fields of science continue to join and contribute to the growth of institution theory, for many of them this activity being an important component of their professional career. A worldwide distributed group of people working in this area is known under the name FLIRTS (`www.informatik.uni-bremen.de/flirts/`).

The aim of this presentation is to guide the reader through the development of institution theory from its seminal paper, included in this anthology, to its current status. We will recall important moments in this process, and discuss the most significant contributions of institution theory. Due to the rather big size of institution theory literature and also due to incompleteness in my knowledge, the omission of important works from this survey and from its references is inevitable. I apologize for all such omissions.

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2. The initial decade

The computing science origins. Institution theory may be the only important trend in universal logic that has emerged from within computing science, the others emerging from logic, and perhaps philosophy. This origin of institution theory may surprise many, since computing science is often blamed for its poor intellectual value. While this perception may be generally correct in average, there are very significant exceptions. In fact, what is now labelled as 'computing science' is hardly a science in the way mathematics or physics are. Some say it is still too young and there was not enough time to coagulate, however one can hear this since 50 years already and probably in the next 50 years too. It may be more realistic to view computing science as a playground were several actors, most notably mathematics, but also logic, engineering, philosophy, sociology, biology, play.

Sometimes an extremely interesting play, which has not only brought significant changes and developments to the actors, but also has revolutionized our scientific thinking in many ways. Institution theory is such an example, the way we think of logic and model theory will never be the same as before.

As the paper included in this anthology shows, the birth of the institution concept came as a response to the population explosion of logical systems in use in specification theory and practice at the time. People felt that many of the theoretical developments (concepts, results, etc.), and even aspects of implementations, are in fact independent of the details of the actual logical systems, that especially in the area of structuring of specifications or programs, it would be possible to develop the things in a completely generic way. The benefit would be not only in uniformity, but also in clarity since for many aspects of specification theory the concrete details of actual logical systems may appear as irrelevant, with the only role being to suffocate the understanding. The first step to achieve this was to come up with a very general formal definition for the informal concept of logical system. Due to their generality, category theoretic concepts appeared as ideal tools. However there is something else which makes category theory so important for this aim: its deeply embedded non-substantialist thinking which gives prominence to the relationships (morphisms) between objects in the detriment of their internal structure. Moreover, category theory was at that time, and continues even now to be so, the mathematical field of the upmost importance for computing science. In fact, it was computing science which recovered the status of category theory, at the time much diminished in conventional mathematical areas. The essay [41] that Joseph Goguen wrote remains one of the most beautiful essays on the significance of category theory for computing science and not only.

The name. In [14] the 'institutions' were called 'languages', but this did not last long. I think every newcomer to the area has wondered about the name 'institution' for the formal mathematical definition of the concept of logical system. It is not straightforward to see the connection between the meaning of this word in common languages and its meaning as an abstract mathematical structure. The fact that mathematics provides other notorious examples of this phenomenon, such as 'group' or even 'category', does not help much. I have heard the following explanation from Joseph Goguen in an Oxford café during my DPhil years. Apparently this name was given half-joke half-truth. At the time (but I think this is still true nowadays), computing scientists had a strong tendency to create social institutions around logical systems. They believed so much in their own logic, that they committed themselves to promoting it, building tools and systems implementing it, starting workshops and conferences devoted to that logic and its applications.

The ACM paper. The paper [14] is perhaps the first properly published work introducing the concept of institution. Moreover, [14] also develops some basics of institution theory but disguised as properties of equational logic. However Goguen and Burstall mention clearly that this was done only to protect some of the audience from the hardship of abstract thinking, since all those properties can be presented at the level of abstract institutions. The first publication focused on the concept of institution is the one included in this anthology ([44] in our list of references), which many consider to be the first "true" institution theory publication, the academic community tends to cite [46] as the seminal paper of this area. This is so because the latter paper is a journal publication and consequently is more elaborated. What puzzles a bit is the rather late publication year of [46]. I remember when during my undergraduate years around mid 1980's, one of my computing science professors in Bucharest noticing how much I was in love with category theory, my passion for model theory, and also my lack of interest for other research trends available in our university at that time, gave me a draft of the institution paper included in this anthology that was circulating in the community. I can say that in sense that event completely shaped my professional future, I immediately realized that my interest fit perfectly the institution theoretic perspective. It is ironic that about six years later I found myself making some small contributions to the last version of that its journal version [46], just before its printing. A prominent German scientist of the same generation also confessed to me that when he first read the same paper he was 'electrized', and one can see now how much institutions are part of his professional achievements. It took journal of ACM not less than 9 years to publish the paper. Why that long? Joseph Goguen explained to me that at some point, after the final acceptance, the chief editor of the journal of ACM was constan tly delaying the actual printing of the paper, so strong was his emotion against the type of thinking promoted by that work. In a way what has happened since then proved him right... During my career I have encountered regularly such kind of emotional reactions, which can be explained by the fear induced by approaches going against the substantialist way of thinking characteristic to the classical western scientific culture.

Myriad of logics as institutions. An important activity of the initial decade of institution theory was to formalize various logics from computing science as institutions. This was mainly motivated by the wish to make use of the specification theoretic results and methods developed at the general level of abstract institutions to the respective logics. There was also another beneficial consequence, the process of formalizing a logic as institution has often led to a conceptual clarification of that logic. The paper included in this anthology presents various fragments of many-sorted first order logic as institutions. Other less conventional logics from computing science have been captured as institutions in a series of papers, some of them never properly published. These includes order sorted algebra in simple form [75] and with sort constraints [86, 63], unified algebras [69], lambda calculus [87], higher order logic with polymorphic types [70], multiple valued [1] and fuzzy logics [40], hidden sorted algebra [42] (and [13] for the order sorted extension), Edinburgh LF [74], or even a model theory for objects, XML, and databases [3].

The beginnings of institution-independent model theory. The definition of the concept of institution provides an ideal meta-mathematical framework for the development of a true abstract model theory free of any commitement to a specific logical system. The main axiom of institutions, the so-called 'satisfaction condition', is inspired by the work of Barwise and others [10, 11], a trend known as 'abstract model theory'. However that trend was only concerned with extensions of conventional logic, hence one may say it is only 'half-abstract'. The true independence from actual logical systems has been achieved by institution theory through a full categorical abstraction of the main logical concepts of signature, sentence, model, and of the satisfaction relation between them. Other general categorical approaches to model theory such as works on sketches [38, 53, 85] or on satisfaction as cone injectivity [4, 5, 6, 58, 57, 56] are also unsatisfactory from the point of view of a true abstract model theory. While the former just develops another language for expressing (possibly infinitary) first order logic realities, the latter considers models as objects of abstract categories but it lacks the multi-signature aspect of institutions given by the signature morphism and the model reducts, which leads to severe methodological limitations. Moreover in these categorical model theory frameworks, the satisfaction of sentences by the models is usually defined rather than being axiomatized.

The first developments of an abstract model theory at the level of abstract institutions belong to Andrzej Tarlecki. Although they were motivated by model theoretic aspects in algebraic specification, they did not have a clear computing science flavour. These works include results about existence of free models of theories [80], axiomatizability of quasi-varieties [81], the initial formulation of the method of diagrams for abstract institutions [81], elements of internal logic such as Boolean connectives and quantifiers for abstract institutions [79]. The work [79] contained also the first formulation of the Craig interpolation property and of a very fundamental form of model amalgamation in abstract institutions.[1] In subsequent developments the latter property gained a crucial role since the majority of computing science or model theory results rely upon this form of model amalgamation. Conventional logic and model theory was unable to realize the importance of this form model amalgamation since most of the actual logics, in fact all of the conventional ones, have this property rather tacitly, and also because of the single-signature orientation in conventional logic and model theory.

[1] Here we refer to a rather common form of model amalgamation across signature morphisms. This is very different from another form of model amalgamation much used in conventional model theory [54], which is across model homomorphisms, and which is much less common.

3. The computing science decade

In the nineties institution theory has witnessed very few model theoretic driven developments without computing science significance. It is mainly for this reason that we call this period the 'computing science decade'.

Foundations of specification languages. During this period institution theory achieved recognition as the most fundamental mathematical structure underlying formal specification, especially algebraic. It has thus become standard to base the definition of specification languages upon logic systems captured as institutions such that all the language constructs are reflected rigorously as mathematical entities in the respective institutions. Moreover, there was the awarness of the importance of a series of model theoretic properties of the underlying institution, as a guarantee for good semantic properties of the respective language, an important example being model amalgamation.

The nineties was the time of the development of the latest generation of algebraic specification languages. CafeOBJ [30, 31] and Maude [19] emerged as direct succesors of the famous OBJ language [48], while CASL [8] was the result of a joint European effort to unify a series of specification frameworks into a new modern language. The definitions of both CafeOBJ and CASL have been strongly based upon institution theory. Due to some errors in the design of rewriting logic [60], Maude failed shortly from having an underlying institution.

Institution-independent specification and programming. The effort to develop specification and programming theory at a generic level independent of any particular institution gave results especially in the area of modularization (structuring) of specification and programs, the so-called specification/programming in-the-large paradigm. These results showed that this paradigm is essentially institution-independent. The work [77] developed the semantics of a set of generic structuring operators at the level of arbitrary abstract institutions, concrete structuring constructs of actual specification languages being derived as combinations of these generic operators. A somehow parallel approach was that of the so-called 'module algebra' of [32], which developed an algebra for software modules applicable to any language rigorously based on an underlying logic captured as institution. The latter work revealed an intimate relationship between the semantical properties of the structuring mechanism and the interpolation properties of the underlying institution. A similar conclusion had emerged from studies on modularization [83, 37, 36] using the so-called 'π-institutions' of [39], an entailment theoretic abstraction of the concept of institution. Moreover, in [12] interpolation properties have shown to represent a crucial condition for a generic institution-independent lifting of complete proof calculi from the level of the basic specifications to that of the structured specifications built with the operators of [77]. At this moment it is important to remind the reader that all this series of institution theoretic developments has been effectively used for the design of languages such as CafeOBJ and CASL, and have also had a strong impact on their associated specification and verification methodologies.

Logic translations. The study of translations between logical systems has an old tradition in logic, and it lies at the core of the universal logic approach since they are a concrete expression of a fundamental philosophical principle relevant for universal logic, that of the co-dependent origination, or interdependency, of logical systems. Therefore, it does not surprise that right from the beginning institution theory has developed concepts of maps between institutions and used them for various purposes such as expressing a logic into another, or for borrowing logical properties or even tools (such as theorem provers). These maps are defined such that they preserve the mathematical structure of the concept of institution. There are two main ways to define such structure preserving maps leading to two main kinds of homomorphisms between institutions: morphisms and comorphisms. Conceptually they are dual to each other, however their use differs a lot. While the former usually expresses a forgetful relationship between a more complex and a simpler institution, the latter is used to formalize embeddings of simpler logics into more complex ones, or to formalize encodings of more complex logics into simpler ones by means of the theories of the simpler logic. While the study of institution morphisms had started with the institution paper included in this anthology, the awarness about comorphisms has developed only gradually (an early reference being [59]). The work [47] is a survey on institution morphisms and comorphisms discussing both structural and methodological aspects related to these concepts of institution mappings. The duality between morphisms and comorphisms has been mathematically shown in [7] in the sense that under an adjunction between the categories of the signatures of the institutions $\mathcal{I}$ and $\mathcal{I}'$, morphisms $\mathcal{I}' \to \mathcal{I}$ and comorphisms $\mathcal{I} \to \mathcal{I}'$ bijectively determine each other. While comorphisms provide the main mathematical notion for developing a systematic theory of doing logic by translation in the sense of the transfer of properties (and even tools, from a more applied perspective) from one logical system to another, for this aim the community has also explored other less established notions for mappings between institutions [61]. Hence doing logic by translation has become a major trend within institution theory, an important pioneering work being [18]. On a more computing science note, institution mappings have been also used to relate formally between specification languages [63].

Logic combination. There was an institution theoretic effort towards this notoriously difficult problem using the concept of *parchment* (see [68]). The so-called 'charters' and 'parchments' have been introduced by the fathers of institution theory, Goguen and Burstall, in [45] as generic technical devices to present institutions, the main axiom (i.e. the satisfaction condition) of institutions being derived at a very general level. While mathematically the charters represent a middle layer between parchments and institutions, the parchments appear as a rather useful concept in its own since they represent meta-level many sorted equational specifications of both the syntax and the semantics of actual logical systems. Later on, the parchment based work [45] inspired other efforts towards the problem of logic combination, such as [16] and [17]. A completely different approach to logic combination is to internalize the features of a specific logic $L1$ to abstract institutions. Then any actual logic $L2$ considered in the role of the abstract institution gives rise to a combination between $L1$ and $L2$. This idea has been realized for possible worlds semantics in [35].

4. The model theory decade

In the third decade all the computing science inspired trends and applications mentioned above have continued. Moreover new applications of institution theory, outside formal specification or declarative programming, have emerged in areas such as ontologies and cognitive semantics [43], concurrency [67], or quantum computing [15]. But probably the most significant developments during this decade were the so-called 'Grothendieck institution' approach to multi-logic heterogeneous specification and the renaissance of a strong model theory activity within institution theory that was not primarily computing science motivated, and which continued at a much deeper level what has been started in the initial decade. Consequently, for the first time institution theoretic papers have been published by non-computing journals such as *J. Symbolic Logic*, *Studia Logica*, or *Logica Universalis*, and institution theory has emerged as an important actor for the universal logic programme. For this reason let us call this decade the 'model theory decade'. The recent monograph [29] includes most of the institution-independent model theory resulting from this activity.

Multi-logic heterogeneous specification. One of the important applications of the institution theoretic approach to logic translations is that of specification languages and frameworks based upon a system of logics rather than upon a single logic. This recent paradigm reflects the understanding that different applications might require different logics, that no single logical system is appropriate for a variety of applications which differ substantially in their nature. One of the earliest works in this direction is [82]. The first specification language that was designed as a multi-logic heterogeneous language was perhaps CafeOBJ [30, 31]. Its semantics was based upon a system of institutions, each of them reflecting a particular specification paradigm, and these were related by a network of embeddings defined formally as comorphisms. A serious problem had emerged: how to make use of the rich existing institution theoretic specification technology for such situation which is not based upon a single underlying institution. One solution, explored in [21], was to extend the institution-independent specification theory, including all basic concepts and results, from a single abstract institution to a system of institutions, i.e. a diagram of institutions, more precisely. However the thinking along these lines has led to a rather different and much more efficient solution, namely that of the 'flattening' of the respective diagram of institutions to a single institution by extending a corresponding construction from category theory [49] to institutions. The resulting concept of *Grothendieck institution* [22, 62] is emerging as the fundamental mathematical structure for the multi-logic heterogeneous specification paradigm. Apart of CafeOBJ, the heterogeneous specification framework with CASL extensions [64] is also based upon the theory of Grothendieck institutions. Quite surprisingly, Grothendieck institutions have been applied to pure model theory, such as for obtaining interpolation results [29].

Doing model theory without concrete structure. The development of model theory at the very general level of abstract institutions is based upon the observation that the most important model theory methods are independent of the conventional first order logic context in which they have originally been developed. This means that all these methods can be formulated and developed at a much more abstract level independent of any particular logical structure. The breakthrough was given by the institution-independent method of ultraproducts [23], which was followed by a rather drastic reformulation in [24] of the institution-independent method of diagrams of [80, 81]. The development of institution-independent saturated model theory [33, 29] came a bit later. These have been used for developing general results about compactness [23], axiomatizability [80, 81, 29], elementary chains [51], interpolation [25, 52], definability [72], completeness [20, 71], generating a big array of novel concrete results in actual unconventional, or even in conventional well studied logics. Moreover, the institution-independent approach to model theory makes the access to highly difficult model theoretic results considerably easier, an example being the Keisler-Shelah isomorphism theorem [33, 29].

Illuminating model theoretic phenomena. The institution-independent approach has lead to the redesign of important fundamental logic concepts and to the clarification of some causality relationships between model theoretic phenomena including the demounting of some deep theoretical preconceptions. One such example is that of interpolation which has been extended to sets of sentences instead of single sentences and to arbitrary commutative squares of signature morphisms instead of the traditional intersection-union squares of signatures. The first extension corrects a traditional misunderstanding about the lack of interpolation properties of logics such Horn clause logic or equational logic. It is the merit of [76] to have proved a Craig interpolation property for sets of sentences in equational logic based upon its Birkhoff-style axiomatizability property, thus revealing a previously unknown cause for interpolation. This idea has been generalized to abstract institutions in [25], thus leading to a myriad of new concrete interpolation results (for fragments of first order logic see also [73]). The second extension of the interpolation concept comes from the practice of algebraic specification which requires interpolation for arbitrary pushout squares of signature morphisms. When interpolation is considered in this way a significant difference between the single and the many sorted logics shows up. The interpolation problem for many sorted first order logic, which stayed for several years as a conjecture, had received a rather elegant solution in [52] as a particular concrete case of a general institution-independent interpolation result. The institution theoretic study of interpolation has also revealed that the Craig-Robinson form of interpolation [78], which stregthens the Craig formulation by adding to the set of the premises a set of 'secondary' premises from the second signature, is actually more appropriate than the traditional Craig formulation. This conclusion is motivated by applications such as definability [72, 29], translation of interpolation [27, 29], modularisation of formal specifications [32, 83, 37], completeness of structured specifications proof calculi [12, 29]. A somehow similar situation happens with (Beth) definability, it can also be extended to arbitrary signature morphisms and formulated more properly in terms of sets of sentences [66, 72], and it can also be obtained as a consequence of Birkhoff-style axiomatizability properties [72]. Another example is given by completeness, which was discovered to have a 'layered' structure as explained below. Both Birkhoff and Gödel-Henkin forms of completeness have been developed at the generic level of abstract institutions in [20] and [71, 50], respectively, by a technique common to both of them, originally developed by [12], and which consists of separating the proof rules and the completeness phenomenon on several layers. In this approach the base layer consists of an institution with a given sound and complete proof system. Since this base layer refers usually to the 'atomic' sentences, its completeness is rather easy to establish in each particular case. The other layers are built on top of the base layer succesively by considering more complex sentences and consequently adding new proof rules and meta-rules. This layered construction is done fully abstractly and the respective completeness results are proved fully generally relative to the completeness of the predecessor level, thus leading especially in the Birkhoff case to a multitude of concrete complete proof calculi for various logics, some of them rather unconventional. Many of these complete proof calculi are new, and quite surprising in that they appear rather remote from the original Birkhoff completeness.

Stratified institutions. This is a recent refinement of the concept of institution which captures uniformly the concept of open formulæ and the concept of models with states (such as possible worlds semantics for modal logics) in a fully abstract setting. Stratified institutions have been developed in [9, 2], however a precursor can be found in [34]. They have already been used to develop a very general version of Tarski's elementary chain theorem applicable to both classical and non-classical (i.e. modal) logics. Stratified institutions also represent a big promise for logic combination, which is one of the great challenges in contemporary logic.

Proof theoretic developments. Although institution theory is primarily model theoretic approach, there have been a proof theory development within institution theory [59, 66, 26, 29, 74] motivated primarily by the foundations for formal verifications. The main goal of the recent approach to proof theory of [66, 26, 29] is to liberate it from the Curry-Howard isomorphism dogma in order to achieve greater simplicity, generality, and harmony with the model theory. Another recent approach to extend institutions with proofs is proposed by [74], its most interesting feature being the conceptual symmetry between the model and the proof theory. Technically speaking, the proof theory of [66, 26, 29], as well as that of [74], follows the proofs-as-arrows idea of categorical logic [55], but it has a much broader range of applications than the latter. Moreover it treats concepts such as implication or quantifiers in a more realistic manner than in categorical logic (for example in categorical logic implication presuposes conjunctions).

Categorical abstract algebraic logic. Although algebraic logic is not a model theoretic approach, we should also mention here the new trend called 'categorical abstract algebraic logic' which develops algebraic logic at the generic abstract level of the π-institutions. The paper [84] is one from a long series of papers on this topic.

The UNILOG connection. Institution theory appears naturally as a major actor in the current universal logic trend, known as UNILOG. Starting with 2005 the UNILOG community is organising world congresses at a rate of each 2-3 years. In each of these congresses it is a custom to organise a competition of papers answering a specific question. In the first congress, held in Switzerland, the institution theory paper [66] failed short to win the first prize for the question 'what is the identity of a logic', but a follow-up paper [65] by the same authors won it at the next congress, held in China, for the question 'what is a logic translation'.

5. Looking to the future

Future is hard to predict, especially in the current climate of scientific research in which theories are developing and trends are changing at an increased speed. Institution theory is already established as the most fundamental mathematical structure for logic based specification theory, and in this sense it will continue to play its foundational role. Moreover institution theoretic ideas will continue to spread in other areas of computing science, however it is difficult to see exactly in which of these and how. In the next period I think the interest for developing model theory at the very general level of abstract institutions, as part of the universal logic trend, will continue to grow. A related area of great interest consists of applying institution-independent model theory to provide a model theory for logical formalisms that do not have a proper one. The new developments such as stratified institutions and the institutional proof theory also represent a big research promise. In longer term I think the most important message given by institution theory is the non-substantialist way of thinking it promotes and its associated top-down methodologies (see [28] for a philosophical essay on this topic).

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Răzvan Diaconescu
Institute of Mathematics “Simion Stoilow” of the Romanian Academy
e-mail: Razvan.Diaconescu@imar.ro