Synthetic scientific report

on the implementation of the project TE 0492 (May 2013 - September 2016)

1 Results

1.1 Articles published or accepted for publication

A1 Andrei Moroianu, **Sergiu Moroianu**, *Ricci surfaces*, Annali della Scuola Normale Superiore di Pisa. Classe di Scienze **14**, no. 4 (2015), 1093-1118

A2 Daniela Anca Măcinic, Ştefan Papadima, **Clement Radu Popescu**, Alexander I. Suciu, *Flat connections and resonance varieties: from rank one to higher ranks*, accepted by Transactions of the American Mathematical Society

A3 Colin Guillarmou, **Sergiu Moroianu**, Frédéric Rochon, *Renormalized volume on the Teichmüller space of punctured surfaces*, accepted by Annali della Scuola Normale Superiore di Pisa. Classe di Scienze

A4 **Sergiu Moroianu**, *The Cotton tensor and Chern-Simons invariants in dimension 3: an introduction*, Buletinul Academiei de Stiinte a Republicii Moldova. Matematica **78**, no. 2 (2015), 3-20

A6 Barbu Berceanu, Daniela Anca Măcinic, Ştefan Papadima, **Clement Radu Popescu**, *On the geometry and topology of partial configuration spaces of Riemann surfaces*, arXiv:1504.04733 preprint, accepted (2016) by Algebraic and Geometric Topology

A7 **Sergiu Moroianu**, *Boundaries of locally conformally flat manifolds in dimensions* $4k$, preprint arXiv:1506.05968, accepted by Indiana University Mathematics Journal

A11 Andrei Moroianu, **Sergiu Moroianu**, *On pluricanonical locally conformally Kähler manifolds*, to appear in International Mathematics Research Notices, doi: 10.1093/imrn/rnw151 (2016)

1.2 Articles submitted for publication

A5 Daniela Anca Măcinic, Ştefan Papadima, **Clement Radu Popescu**, *Modular equalities for complex reflection arrangements*, arXiv:1406.7137 preprint, submitted

A8 **Sergiu Moroianu**, *Convexity of the renormalized volume of hyperbolic 3-manifolds*, arXiv:1503.07981 preprint, submitted

A9 **Gabriel Bădiţoiu**, *Integrable systems and Connes-Kreimer renormaliation*, submitted

A10 **Dorin Cheptea**, *Jacobi diagrams on surfaces and quantum invariants*, submitted

1.3 Books

C1 Jean-Pierre Bourguignon, Oussama Hijazi, Jean-Louis Milhorat, Andrei Moroianu, **Sergiu Moroianu**, *A Spinorial Approach to Riemannian and Conformal Geometry* (458 pages), EMS Monographs in Mathematics (2015), ISBN: 978-3-03719-136-1

Journal versions will be accessible from the grant's webpage: http://www.imar.ro/~dcheptea/grantul_0492.html

2 Resume of some of the results and research problems

Within the project PN-II-RU-TE-2012-3-0492, a series of problems were investigated and the following results were obtained.

2.1 Ricci surfaces

In the paper [**A1**], Ricci sufaces were studied, i.e. surfaces with a Riemann metric whose Gaussian curvature satisfies the equation $K\Delta K + g(dK, dK) + 4K^3 = 0$. Ricci-Curbastro (1890) have shown that every minimal surface in $\mathbb{R}^3$ satisfies this constraint; reversely, if $K \neq 0$ then the surface can be (locally) isometrically embedded in $\mathbb{R}^3$. The main result in [**A1**] shows that the same is true also around the points where K vanishes. The method used in investigate Ricci surfaces is based on the description of local embeddings by the existence of a nonzero harmonic spinor of constant norm. Finding such a spinor reduces to a problem in analysis: under what conditions a real-valued function on the unit disk is the square of the norm of a holomorphic function? A necessary condition is the log-harmonicity of the function. Our technical result, based on ideas from potential theory, shows that this condition is also suficient. The problem undertaken in this paper is closely related to the existence of minimal surfaces in manifolds of negative constant curvature, studied for example by [1]. Such manifolds of dimension 3, complete but with infinite volume, are studied because of geometrization of hyperbolic cobordisms between their ideal at infinity boundary components.

2.2 Renormalized volume of hyperbolic manifolds

The renormalized volume is a functional on the space of hyperbolic manifolds of finite geometry, which furnish a Kähler potential on the Teichmüller space. In the paper [**A8**] we show that near a metric with ideal boundary of product type, the renormalized volume of the 3-dimensional manifold has positive Hessian.

In the paper [**A3**] we study this renormalized volume for groups that degenerate towards groups with parabolic subgroups of rank 1. We show that the renormalized volume is continuous at the boundary of the Teichmüller space, if we control the degeneration parameters so that the Dehn rotation angle is upper bounded by the length of the simple closed geodesic that is shrunk to a point.

2.3 Boundary of locally conformally flat manifolds of dimension $4k$

In the paper [**A7**] we show that the eta invariant of the signature operator on an oriented compact Riemann manifold M of dimension $4k-1$ is an obstruction to the existence of a locally conformally flat metric on an oriented compact manifold X, whose boundary is M.

2.4 Pluricanonical locally conformally Kähler manifolds

In the paper [**A11**] we give a short proof of the fact that compact pluricanonical locally conformally Kähler manifolds have parallel Lee form, i.e. they are Vaisman.

2.5 Flat connections and resonance varieties

A finitely generated group π (examples are fundamental groups of link complements, fundamental groups of hyperplane complements, or Torelli groups for $g \geqslant 3$) can be studies by analyzing the representation variety $\mathrm{Hom}(\pi, G)$, where G is a linear algebraic group. These varieties are complicated (Kapovich-Millson universality theorem), but an algebraic analogue is the set of flat connections $\mathscr{F}(A, \mathfrak{g}) = \{\omega \in A^1 \otimes g \mid \partial\omega + \frac{1}{2}[\omega, \omega] = 0\}$, where for a commutative differential graded algebra (cdga) $A = (A^{\bullet}, d)$ and a Lie algebra $\mathfrak{g}$, the tensor product $A \otimes \mathfrak{g}$ has a graded differential Lie algebra structure with the Lie bracket $[\alpha \otimes x, \beta \otimes y] = \alpha\beta \otimes [x, y]$ and differential $\partial(\alpha \otimes x) = d\alpha \otimes x$. If the cdga A is connected (i.e. $A^0 = \mathbb{C} \cdot 1$) and $\mathfrak{g} = \mathbb{C}$, then $\mathscr{F}(A, \mathbb{C}) = H^1(A)$.

Given a connected cdga$A$ of finite $q-$type ($A^{\leqslant q}$ is finite dimensional) and a linear representation $\theta\colon \mathfrak{g} \to \mathfrak{gl}(V)$, the tensor product $A \otimes V$ becomes a cochain complex with the differential $d_\omega = d \otimes \mathrm{id}_V + \mathrm{ad}_\omega$, which depends on ω and θ. If $\mathfrak{g}$ and V are finite dimensional then the sets, called resonance varieties,

$$\mathscr{R}^i_r(A, \theta) = \{\omega \in \mathscr{F}(A, \mathfrak{g}) \mid \dim_{\mathbb{C}} H^i(A \otimes V, d_\omega) \geqslant r\}$$

are Zariski closed in $\mathscr{F}(A, \mathfrak{g})$ and form a filtration of the set of flat connections.

Suppose that A is 1-finite and $\mathfrak{g}$ is finite dimensional. Consider $\mathscr{F}^1(A, \mathfrak{g}) \subseteq \mathscr{F}(A, \mathfrak{g})$ consisting of tensors $\eta \otimes g$ for which $d\eta = 0$. For a representation $\theta\colon \mathfrak{g} \to \mathfrak{gl}(V)$ consider $\Pi(A, \theta) \subseteq \mathscr{F}^1(A, \mathfrak{g})$ the subvariety of the tensors which satisfy also the equality $\det(\theta(g)) = 0$. In the paper [**A2**] Radu Popescu and his coathors have proved:

Theorem 1. *Let $\omega = \eta \otimes g$ be an arbitrary element of $\mathscr{F}^1(A, \mathfrak{g})$. Then ω belongs to $\mathscr{R}^k_1(A, \theta)$ if and only if there is an eigenvalue λ of $\theta(g)$ such that $\lambda\eta$ belongs to $\mathscr{R}^k_1(A)$. Moreover,*

$$\Pi(A, \theta) \subseteq \bigcap_{k: H^k(A) \neq 0} \mathscr{R}^k_1(A, \theta).$$

Let π a finitely-generated nilpotent group and G either semisimple of rank 1, or one of its Borel subgroups. Given a rational representation $\iota\colon G \to \mathrm{GL}(V)$, with the tangent map θ, we show that $\mathscr{F}(A, \mathfrak{g}) = \mathscr{F}^1(A, \mathfrak{g})$, where $A = \mathcal{C}^{\bullet}(\mathfrak{n})$ is the Chevalley-Eilenberg cochain algebra of the Malcev-Lie algebra $\mathfrak{n}$ associated to π. Moreover

$$\mathscr{R}^k_1(\mathcal{C}^{\bullet}(\mathfrak{n}), \theta) = \Pi(\mathcal{C}^{\bullet}(\mathfrak{n}), \theta)$$

if $k \leqslant \dim \mathfrak{n}$ and is empty otherwise.

Let A be a connected and 1-finite cdga for which the variety $\mathscr{R}^1_1(A)$ decomposes as a finite union of linear subspaces, and let θ be a finite dimensional representation of a finite dimensional Lie algebra. We obtained [**A2**] the following result:

Theorem 2. *Suppose $\mathscr{R}^1_1(A) = \bigcup_{C \in \mathscr{C}} C$, a finite union of linear subspaces. For each $C \in \mathscr{C}$, let A_C denote the sub-cdga of the truncation $A^{\leqslant 2}$ defined by $A^1_C = C$ and $A^2_C = A^2$. Then, for any Lie algebra $\mathfrak{g}$,*

$$\mathscr{F}(A, \mathfrak{g}) \supseteq \mathscr{F}^1(A, \mathfrak{g}) \cup \bigcup_{0 \neq C \in \mathscr{C}} \mathscr{F}(A_C, \mathfrak{g}), \tag{1}$$

where each $\mathscr{F}(A_C, \mathfrak{g})$ is Zariski-closed in $\mathscr{F}(A, \mathfrak{g})$. Moreover, if A has zero differential, A^1 is non-zero, and $\mathfrak{g} = \mathfrak{sl}_2$ or $\mathfrak{sol}_2$, then (1) holds as an equality, and, for any θ,

$$\mathscr{R}^1_1(A, \theta) = \Pi(A, \theta) \cup \bigcup_{0 \neq C \in \mathscr{C}} \mathscr{F}(A_C, \mathfrak{g}). \tag{2}$$

For a 1-formal 1-finite space X and $A = (H^\bullet(X, \mathbb{C}), d = 0)$ it is known that the resonance variety in rank 1, $\mathscr{R}^1_1(A)$ is linear. Another example of a space with the above mentioned properties is the classifying space X_Γ of a right-angled Artin group π_Γ. For the computation of the resonance variety in rank 1 case, the model of the space is the cdga $A_\Gamma = (H^\bullet(\pi_\Gamma, \mathbb{C}), d = 0)$. For this we can apply Theorem 2. For π_Γ a right-angled Artin group we have the following irreducible decomposition:

$$\mathscr{F}(A_\Gamma, \mathfrak{sl}_2) = \bigcup_{\mathsf{W} \subseteq \mathsf{V}} S_{\mathsf{W}}$$

where W runs through the subsets of vertex set of Γ, maximal with respect to an order $\preccurlyeq$ defined in terms of the connected components of the induced subgraph Γ_{W}. S_{W} is a certain combinatorially defined, closed subvariety of $\mathbb{C}^{\mathsf{W}} \otimes \mathfrak{sl}_2$.

2.6 Modular equalities for complex reflection arrangements

In the paper [**A5**], Radu Popescu in collaboration with A. Măcinic and Ş. Papadima have computed the combinatorial Aomoto-Betti numbers $\beta_p(\mathcal{A})$ for complex reflection arrangements.

For a hyperplane arrangement $\mathcal{A}$, the complement $M_{\mathcal{A}}$ has the homotopy type of a finite CW-complex with torsion-free homology, and in particular $H_1(M_{\mathcal{A}}, \mathbb{Z}) = \mathbb{Z}^n$. The resonance variety in degree i and depth q over a field k is defined as:

$$\mathscr{R}^i_q(M_{\mathcal{A}}, \Bbbk) = \{\sigma \in H^i(M_{\mathcal{A}}, \Bbbk) \mid \dim_{\Bbbk} H^i(H^\bullet(M_{\mathcal{A}}, \Bbbk), \sigma\cdot) \geqslant q\}$$

where $\sigma\cdot$ represents the left multiplication with σ in the cohomology ring. For $M = M_{\mathcal{A}}$, and $\Bbbk = \mathbb{F}_p$ (p prime), denote by $\sigma_p \in H^1(M, \mathbb{F}_p)$ the diagonal cohomology class, equal to 1 with respect to the distinguished $\mathbb{Z}$-basis given by the meridians around the hyperplanes. The modulo p Aomoto-Betti number is

$$\beta_p(M) \coloneqq \dim_{\mathbb{F}_p} H^1(H^\bullet(M, \mathbb{F}_p), \sigma_p\cdot)$$

Theorem 3. *For a complx rflaction arrangement $\mathcal{A}$ of rank at least 3, the following hold.*

1. *If $p > 3$, then $\beta_p(\mathcal{A}) = 0$.*
2. *$\beta_2(\mathcal{A}) \neq 0 \Leftrightarrow \beta_2(\mathcal{A}) = 2 \Leftrightarrow \mathcal{A}$ supports a 4-net $\Leftrightarrow \mathcal{A}$ is the Hessian arrangement.*
3. *The only cases when $\beta_3(\mathcal{A}) \neq 0$ are: $\mathcal{A}(m, 1, 3)$ with $m \equiv 1 \ (mod\, 3)$, where $\beta_3 = 1$; $\mathcal{A}(m, m, 3)$ with $m \geqslant 2$, where $\beta_3 = 1$ if $m \not\equiv 0 \ (mod\, 3)$ and otherwise $\beta_3 = 2$; $\mathcal{A}(m, m, 4)$, where $\beta_3 = 1$.*
4. *In particular, $\beta_p(\mathcal{A}) \leqslant 2$, for all primes p.*

2.7 Partial configuration spaces of Riemann surfaces

The study of topological properties of a CW-complex X, q-finite cu $q \geqslant 1$, can be undertaken via the representation variety $\mathrm{Hom}(\pi, G)$, where $\pi = \pi_1(X)$ and G is a linear algebraic group. The algebraic counterpart fot eh representation variety, introduced by Dimca-Papadima-Suciu [2009] is the set of flat connections, which has a filtration given by resonance varieties (see above).

The concepts and the results obtained in [**A2**] with respect to the variety of flat connections and the resonance subvarieties of rank greater than 1 were applied in [**A6**] (2015):

Let Γ be a finite simple graph with the set of vertices V and the set of edges E, and Σ be a space. The *partial configuration space* of type Γ on the space Σ is given by

$$F(\Sigma, \Gamma) = \{z \in \Sigma^{\mathsf{V}} \mid z_i \neq z_j, \quad \text{for all} \quad ij \in \mathsf{E}\}.$$

For the case when $\Gamma = K_n$, the complete graph with n vertices, $F(\Sigma, \Gamma)$ is the classical space of configurations of ordered points in Σ. If $\Sigma = \Sigma_g$, a compact Riemann surface og genus g, denote the space $F(\Sigma, \Gamma)$ by $F(g, \Gamma)$ and by $P(g, \Gamma) = \pi_1(F(g, \Gamma))$, the fundamental group of the partial configuration space. We note that this represents the natural generalization of the pure braid group case, which corresponds to the graph $\Gamma = K_n$ and $\Sigma = \mathbb{C}$.

In the general case, when $\Gamma \neq K_n$, there is **no** Fadell-Neuwirth fibration theorem as it happens for the pure braid group. Nevertheless, in this paper we calculate invariants of the parcial configuration space.

Regarding Σ_g as a smooth complex curve of genus g, $F(g, \Gamma)$ becomes a quasi-projective variety (as above - irreducible smooth quasi-projective complex variety).

For every genus g we can easily construct certain admissible maps of general type on $F(g, \Gamma)$. These maps are associated to embeddings of complete graphs in Γ.

For $g \geqslant 2$ there are embeddings $K_1 \hookrightarrow \Gamma$ which give the maps $f_i : F(g, \Gamma) \to \Sigma_g$ induced by the projection specified by the corresponding vertex $i \in \mathsf{V}$.

For $g = 1$ there are enbeddings $K_2 \hookrightarrow \Gamma$ which give the maps $f_{ij} : F(1, \Gamma) \to \Sigma_1 \backslash \{0\}$ given by the projection corresponding to the edge $ij \in \mathsf{E}$, followed by the substraction map on the elliptic curve Σ_1.

For $g = 0$ there are embeddings $K_4 \hookrightarrow \Gamma$ and $f_{ijkl} : F(0, \Gamma) \to \mathbb{P}^1 \backslash \{0, 1, \infty\}$ is composition of the cross-ratio and the projection associated to the vertex set of Γ given by the embedding $K_4 \hookrightarrow \Gamma$.

We obtained [**A6**]:

Theorem 4. *A complete set of representatives for $\mathscr{E}_{F(g,\Gamma)}$ is given by admissible maps for general type described above.*

Hence all the admissible maps of general type on $F(g, \Gamma)$ are the ones described above.

Let X be a quasi-projective variety $X = \overline{X} \backslash D$, where $\overline{X}$ is a smooth compactification constructed by Dupont, obtained by adding at infinity of a hypersurface arrangement D in $\overline{X}$. Dupont has constructed a Gysin model $A^\bullet(\overline{X}, D)$, associated to the pair $(\overline{X}, D)$, which is finite and has naturality properties. By a theorem of Dimca-Papdima [2014] ot follows that the model $A^\bullet$ determines $\mathscr{E}_X$, which is in 1-1 correspondence with the positive-dimensional irreducible components through the origin for $\mathscr{R}^1_1(A)$. For $X = F(g, \Gamma)$ we shall take $\overline{X} = \Sigma_g^{\mathsf{V}}$ and $D_\Gamma = \bigcup_{ij \in \mathsf{E}} \Delta_{ij}$ the union of all the diagonals associated to the edges of the graph. Let $f : F(g, \Gamma) \to S = \overline{S} \backslash F$ be the admissible maps in theorem 4, where $\overline{S} = \Sigma_g$ and $F \subseteq \overline{S}$ is a finite subset (in particular, a hypersurface arrangement in $\overline{S}$). In [**A6**] we obtained:

Theorem 5. *In the above context there is a regular extension of f, $\overline{f} : (\overline{X}, D) \to (\overline{S}, F)$, for all $f \in \mathscr{E} := \mathscr{E}_{F(g,\Gamma)}$, where D is a hypersurface arrangement in $\overline{X}$ with complement $F(g,\Gamma)$, which induces* cdga *maps between the Gysin models, $f^* : A^\bullet(\overline{S}, F) \to A^\bullet(\overline{X}, D)$, with the property that*

$$\mathscr{F}(A^\bullet(\overline{X}, D), \mathfrak{g}) = \mathscr{F}^1(A^\bullet(\overline{X}, D), \mathfrak{g}) \cup \bigcup_{f \in \mathscr{E}} f^* \mathscr{F}(A^\bullet(\overline{S}, F), \mathfrak{g}) \quad \textit{for } \mathfrak{g} = \mathfrak{sl}_2 \textit{ or } \mathfrak{sol}_2\,, \tag{3}$$

and

$$\mathscr{R}^1_1(A^\bullet(\overline{X}, D), \theta) = \Pi(A^\bullet(\overline{X}, D), \theta) \cup \bigcup_{f \in \mathscr{E}} f^* \mathscr{F}(A^\bullet(\overline{S}, F), \mathfrak{g})\,, \tag{4}$$

for any finite-dimensional representation $\theta : \mathfrak{g} \to \mathfrak{gl}(V)$.

2.8 Functoriality and integrability of Lax type equations in the context of Connes-Kreimer renormalization [A9]

In the paper [6], prior to this grant, Gabriel Bădiţoiu and Steven Rosenberg have constructed a Lax pair equation $\frac{dL}{dt} = [L, M]$ associated to Connes-Kreimer-Birkhoff factorization oa commutative, connected, graded Hopf algebra, have shown that the flow of this equation preserves the locality property of counter-terms and have made the connection with the flow of the renormalization group. There are 2 approaches: (i) one considers the double Lie algebra $\mathfrak{g} \oplus \mathfrak{g}^*$ associated to a finite-dimensional truncation $\mathfrak{g}$ of the Lie algebra of infinitesimal characters of a graded Hopf algebra (the Lie bracket on the Lie subalgebra $\mathfrak{g}^*$ is the abelian one) and one studies the integrability of the flow of the Lax equation in particular cases; and (ii) one constructs the Lax equation on the entire (infinite-dimensional) Lie algebra of infinitesimal characters, and obtains a Lax equation for the beta function of the quantum field theory.

The integrability of the Lax pair equations associated to semisimple Lie algebras by the Adler-Kostant-Symes theorem is well understood (cf.Reyman si Semenov-Tian-Shansky [8]), but the nilpotent case is only little studied.

As part of this grant, extending results from [6], Gabriel Baditoiu has constructed examples of 3-step or 4-step nilpotent Lie algebras (of infinitesimal characters associated to some finite-dimensional Hopf algebras of roots), for which the associated Lax equations are integrable, respectively Hamiltonian. To prove integrability integrals of motion were calculated explicitly. Using the Mishchenko-Fomenko conjecture, we get that a finite-dimensional truncation of the Lax pair equation for the beta-function is completely integrable under certain conditions. These results are included in [**A9**].

Gabriel Baditoiu and Steven Rosenberg study the 'functoriality' of the Lax equation when one changes the Hopf algebra (quantum field theory in the context of Connes-Kreimer renormalization). In the simpler case (i) of finite-dimensional truncation we use the Etingof-Kazhdan functor from the category of Lie bialgebras to the category of quantized enveloping algebras [7]. For a Lie bialgebra morphism $\alpha : \mathfrak{g} \to \mathfrak{h}$ we obtain a Hopf algebra morphism between quantized universal enveloping algebras of double Lie algebras $\tilde{\alpha} : \mathscr{U}(\mathfrak{g} \oplus \mathfrak{g}^*)[[h]] \to \mathscr{U}(\mathfrak{h} \oplus \mathfrak{h}^*)[[h]]$, which induces a Lie algebra morphism between the Lie algebras of infinitesimal characters of the duals of these Hopf algebras $\partial\alpha^* : \partial(Char(\mathscr{U}(\mathfrak{g} \oplus \mathfrak{g}^*)^*[[h]])) \to \partial(Char(\mathscr{U}(\mathfrak{h} \oplus \mathfrak{h}^*)^*[[h]]))$. Identifying $\partial(Char(\mathscr{U}(\mathfrak{g} \oplus \mathfrak{g}^*)^*[[h]])) = \mathfrak{g} \oplus \mathfrak{g}^* \oplus \mathbb{C}$ and $\partial(Char(\mathscr{U}(\mathfrak{h} \oplus \mathfrak{h}^*)^*[[h]])) = \mathfrak{h} \oplus \mathfrak{h}^* \oplus \mathbb{C}$ we get a Lie algebra morphism $\partial\alpha^* : \mathfrak{g} \oplus \mathfrak{g}^* \oplus \mathbb{C} \to \mathfrak{h} \oplus \mathfrak{h}^* \oplus \mathbb{C}$, which sends a flow $L(t)$ of a Lax equation on $\mathfrak{g} \oplus \mathfrak{g}^* \oplus \mathbb{C}$ to a flow $\partial\alpha^*(L(t))$ of another associated Lax equation on $\mathfrak{h} \oplus \mathfrak{h}^* \oplus \mathbb{C}$.

2.9 Jacobi diagrams on surfaces and quantum invariants [A10]

W. M. Goldman (1986) has defined a Poisson-Lie algebra structure on the symmetric algebra of the K-module generated by the set $\hat{\pi}$ of free homotopy classes of loops $S^1 \to \Sigma$ on a surface Σ, as well as deformations of that in the category of Poisson-Lie algebras. In 1988 and 1991, V. Turaev has constructed skein quantizations of these Poisson algebras. In 1996 and 1998, J.E. Andersen, J. Mattes and N. Reshetikhin [2, 3], generalizing Turaev's works on Goldman bracket, have constructed the Poisson algebra of chord diagrams on an arbitrary surface, and, using an (abstract) universal finite-type invariant of links in$\Sigma \times [0,1]$ (Σ is compact with free fundamental group), have built a deformation quantization of the Poisson algebra.

In the paper [**A10**], we introduce and study the notion of Jacobi diagrams on arbitrary surfaces, which generalizes the notion of chord diagrams on surfaces. This generalization in the case of $\Sigma = \mathbb{R}^2$ is the central algebraic-combinatorial object in the theory of the Le-Murakami-Ohtsuki (LMO) functor, one of the two manifestations of quantum (Witten-Reshetikhin-Turaev) invariants of links, 3-manifolds and (2+1)-cobordisms. (See, for example, [4, 5].) Using Jacobi diagrams instead of chord diagrams, and the LMO functor instead of the ad-hoc invariant of Andersen-Mattes-Reshetikhin, in [P2] we obtain the constructions of Andersen-Mattes-Reshetikhin in a more systematic and general way, as well as new consequences. intrun mod mai sistematic si mai general, precum si consecinte noi.

2.10 Volume conjecture, Geer-Patureau invariants, and Turaev-Viro $6j$-symbols

Cristina Anghel was hired into the grant team by competition for the period October 2013 - September 2016. In September - mid December 2014, January - June, September-November 2015, and January - mid June 2016 she was away at Universite Paris 7; during those periods she was not supported from the grant, but we have continued to collaborate mathematically by email. While on this grant, Cristina Anghel has finished 3 master programs: at the University of Bucharest (the master thesis "Volume conjecture for knots and links" was co-supervised by Dorin Cheptea), at Scoala Normala Superioara Bucuresti, and at Universite Paris 7. Since September 2015 she is a PhD student.

On May 6, 2015, and on June 24, 2016 she delivered talks at the Topology Seminar at IMAR ("Heegard-Floer homology: definitions, examples, basic results", respectively "Volume conjecture for knots and links"). She has also delivered talks in the Groupe de travail "Topologie Algbrique et TQFT" at Universite Paris 7, as well as at the "ECSTATIC" conference, Imperial College of London.

She knows very well differential topology, low dimensional topology, complex and hyperbolic geometry, Hodge theory, Riemann surfaces, representation theory. He has studies the basic books and papers on Reshetikhin-Turaev invariants, Heegaard-Floer homology, and volume conjecture, the next natural step being to directly approach research problems (2015-).

In 2006 and 2007, N. Geer and B. Patureau-Mirand have introduced a class of link invariants, in several variables, renormalized, which come from Lie superalgebras of type I, or from $sl(2|1)$, and later (2013) together with Turaev have constructed them for any ribbon category, not necessarily coming from an algebra. Also Geer and Patureau have shown that their invariants for a family of $sl(m|n)$ super-modules contain Kashaev's invariants. Kashaev's invariants (1995) where introduced together with the first statement of the volume conjecture. In 2001, H. Murakami and J. Murakami have proved that Kashaev's invariants are a particular specification of the coloered

Jones polynomial, And hence the volume conjecture was reformulated in its modern version. It related the colored Jones polynomial of a link K with the volume of the link complement $S^3\backslash K$, i.e. an object from quantum topology with an object from hyperbolic geometry:

$$\lim_{N\to\infty}\frac{\log|J_N(K,e^{2\pi i/N})|}{N}=\frac{1}{2\pi}vol(S^3\backslash K)$$

Cristina Anghel's research activity is focused on the following 3 mutually connected problems:

- Cristina Anghel and Dorin Cheptea study a possible new approach to volume conjecture based on the technique of cluster algebras, which was introduced recently to hyperbolic geometry by Fomin-Shapiro-D. Thurston (2008) and Nagao-Terashima-Yamazaki (2011)
- In 2015, while in Paris Cristina Anghel met Nathan Geer (Utah State University, USA) in visit there, and as a result they have started to work on a possible extension of Geer-Patureau invariants for 3-dimensional manifolds
- Cristina Anghel and Christian Blanchet (Universite Paris 7) study the problem of modified $6j$-symbols for the case of the superalgebra $sl(2|1)$, aiming to construct Turaev-Viro type invariants of 3-manifolds

3 Activities

3.1 Invited guests

- Frederic Rochon (Universite du Quebec a Montreal), July 2-5, 2013
- Colin Guillarmou (Ecole Normale Superieure, Paris), November 4-9, 2013
- Michel Pocchiola (Institut de Mathematiques de Jussieu, Paris), November 18-22, 2013
- Gwenael Massuyeau (Universite de Starsbourg & CNRS), November 18-22, 2013
- Florin Damian (Universitatea de Stat din Moldova, Chişinău), November 25-27, 2013
- Dan Burghelea (Ohio State University, Columbus, OH), September 6-14, 2014
- Andrei Moroianu (CNRS & Universite de Versailles-St Quentin, France), April 12-20, 2015
- Jean-Stephane Dhersin (Universite Paris 13), October 24 - November 2, 2015
- Mylene Maïda (Universite Lille 1), October 29 - November 2, 2015
- Jin-ichi Itoh (Kumamoto University, Japan) November 26-30, 2015
- Andrei Moroianu (CNRS & Universite de Versailles-St Quentin, France), November 25 - December, 2015
- Colin Guillarmou (Ecole Normale Superieure, Paris), April 17-22, 2016
- Andrei Moroianu (CNRS & Universite de Versailles-St Quentin, France), March 23-31, 2016

The quests have undertaken research collaboration and have delivered formal talks. More details are present on the grant's webpage.

3.2 Participations

- Cristina Anghel has participated in October 6-12, 2013 in the school "Second Erlangen Fall School on Quantum Geometry", Erlangen, Germany
- Cristina Anghel has participated in May 21-23, 2014 in the "Fourth Workshop for Young Researchers in Mathematics", Constanta, Romania
- Cristina Anghel has participated in June 29 - July 7, 2014 in the "Young Topologists Meeting", Copenhagen, Denmark
- Clement Radu Popescu has participated in October 13-17, 2014 in the conference "Tresses et Arithmetique", CIRM, Luminy, France
- Gabriel Băditoiu has participated in June 30 - July 25, 2014 in the "Clay Mathematics Institute Summer School 2014 Periods and Motives: Feynman amplitudes in the 21st century", ICMAT, Madrid, Spain
- Clement Radu Popescu has participated in May 20-24, 2015 in the "Workshop for Young Researchers in Mathematics", Constanta, Romania (talk title "Resonance varieties and nilpotent Lie algebras")
- Clement Radu Popescu has participated in June 26 - July 1, 2015 in "The Eighth Congress of Romanian Mathematicians", Iasi, Romania (talk title "Flat connections and resonance varieties of rank larger than 1")
- Clement Radu Popescu has participated in July 26 - August 1, 2015 in the summer school and conference "Mapping class groups, 3- and 4-manifolds", Cluj-Mapoca, Romania
- Sergiu Moroianu has participated in July 1-5, 2015 in the "MITRE 2015" conference, Chisinau, Moldova (talk title "Boundaries of locally conformally flat $4k$ manifolds")
- Cristina Anghel has participated in July 4-11, 2015 in the "Young Topologists' Meeting", Lausanne, Switzerland (talk title "Renormalized quantum dimension and Multivariable invariants for links")
- Cristina Anghel has participated in July 26 - August 2, 2015 in the summer school and conference "Mapping class groups, 3- and 4-manifolds", Cluj-Mapoca, Romania
- Radu Popescu has participated in May 18-21, 2016 in the "Workshop for Young Researchers in Mathematics", Constanta, Romania (talk title "Flat connections and quasi-projective varieties")
- Dorin Cheptea has participated in May 18-21, 2016 in the "Workshop for Young Researchers in Mathematics", Constanta, Romania (talk title "Jacobi diagrams on surfaces and quantum invariants")
- Sergiu Moroianu has participated in May 18-21, 2016 2016 in the "Workshop for Young Researchers in Mathematics", Constanta, Romania, where he organized a special session of for the anniversary of the Romanian Academy

- Sergiu Moroianu was invited in May 8-13, 2016 at the University of Fribourg, Switzerland for scientific collaboration; on May 10, 2016 he delivered a talk "Renormalized volume in hyperbolic geometry" in the Colloquium of the University of Fribourg
- Sergiu Moroianu has participated in June 10-11, in the "Geometry and PDEs Workshop", Universitatea de Vest, Timişoara, Romania, (talk on June 10, 2016)

Also, financed from other sources (organizers) but connected to grant related research:

- Gabriel Baditoiu has participated in March 1-14, 2015 in the conference "The interrelation between mathematical physics, number theory and noncommutative geometry", Erwin Schrödinger International Institute for Mathematical Physics, Vienna (where G.B. has worked with Steven Rosenberg on naturality property of Lax type equations, focusing on studying Rota-Baxter algebras)

More details on the grant's webpage.

3.3 IMAR seminar initiated within the grant

"Two- and three-dimensional manifolds" http://www.imar.ro/~dcheptea/seminar23dim.html

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