

# Scientific Report for Phase I of the Research Grant 'Special Geometries and Associated Structures'

Liana David

This report describes the scientific activity of the team members in the research grant 'Special Geometries and Associated Structures', project code PN-III-ID-P4-PCE-2016-0019, in Phase I of the grant, in the period July 12th - December 30th, 2017.

The research team is composed by: CS I Dr. Liana David (Project Director), Associate Prof. Dr. Monica Aprodu (key-member of the project team), CS III Dr. Gabriel Baditoiu, CS I Dr. Radu Pantilie, CS II Dr. Costin Vilcu, and master student Ionut-Alexandru Hurjui.

## 1 Papers submitted for publication

V. Cortes, L. David: *Twist, elementary deformation, and the KK-correspondence in generalized complex geometry*, arxiv:1706.05516, 57 pages.

## 2 Research activity of the team members

**Monica Aprodu** focussed her research on two directions:

1. The existence of harmonic homotopy classes on certain complex hermitian surfaces.
2. The investigation of invariants on Riemannian polyhedra.

**Liana David**, together with Vicente Cortes, developed the preprint *Twist, elementary deformation and the KK-correspondence in generalized complex geometry*, arxiv:1706.05516. In this preprint we define the operations of conformal change and elementary deformation in the setting of generalized complex geometry. Then we apply Swann's twist construction to generalized (almost) complex and Hermitian structures obtained by these operations and establish necessary and sufficient conditions for the Courant integrability of the resulting twisted structures. In particular, we associate to any appropriate generalized Kähler manifold  $(M, G, \mathcal{J})$  with a Hamiltonian Killing vector field a new generalized Kähler manifold, depending on the choice of a pair of non-vanishing functions and compatible twist data. We study this construction when  $(M, G, \mathcal{J})$  is (diagonal) toric, with emphasis on the four-dimensional case. In particular, we apply it to deformations of the standard flat Kähler metric on  $\mathbb{C}^n$ , the Fubini-Study Kähler metric on  $\mathbb{C}P^2$  and the so called admissible Kähler metrics on Hirzebruch surfaces. As a further application, we recover the KK (Kähler-Kähler) correspondence, which is obtained by specializing to the case of an ordinary Kähler manifold.

The researchers Gabriel Baditoiu, Radu Pantilie and Costin Vilcu joined the grant team on November 16th, 2017. The master student Ionut-Alexandru Hurjui joined the grant team on

December 1st, 2017. Due to the short time since their hiring, they do not report new research results obtained in the framework of this grant. We describe below the research directions they are addressing.

**Gabriel Baditoiu** is studying the problem of connecting some Einstein metrics through Ricci flow on certain homogeneous spaces.

**Radu Pantilie** is studying the ro-quaternionic manifolds and, also, the twistorial maps and harmonic morphisms that exist between these. This way, a better understanding of these objects and morphisms will be obtained through a study of relevant particular classes, thus obtaining new such natural constructions.

**Costin Vilcu** is studying critical points for distance functions on Riemann and Alexandrov surfaces.

**Ionut-Alexandru Hurjui** is studying the book *Riemannian Geometry* by Manfredo Perdigao do Carmo, thus forming his differential geometry culture. He also participates to the 'Geometry' seminarul at IMAR (the host institution for the project).

### 3 Talks at workshops

We report the following talks at workshops related to the project theme.

**Monica Aprodu**, Curvature on polyhedra, Workshop *Mathematics, Computer Science and Applications*, Galati, 13 - 15 December, 2017.

**Costin Vilcu**, Folding and gluing in convex geometry, *International Symposium on Discrete and Convex Geometry*, Hebei Normal University, Shijiazhuang, China, 15 - 18 December, 2017.

**Costin Vilcu**, From convex surfaces to Alexandrov surfaces, *International Symposium on Discrete and Convex Geometry*, Hebei Normal University, Shijiazhuang, China, 15 - 18 December, 2017.

### 4 Visits abroad

The following visits abroad took place, for possible research collaborations on the project topics.

**Monica Aprodu** visited, between September 27th - October 1st, 2017, *Departimento di Matematica* at *University of Genova* (Italy).

**Liana David**: visited, during the period October 23rd - November 11th, 2017, *University of Mannheim* (Germany), *Lehrstuhl fuer Mathematik VI*, in order to work with Claus Hertling on meromorphic connections.

**Radu Pantilie** visited, between December 3rd - December 16th, 2017, *Laboratoire de Mathématiques de Bretagne Atlantique* at *University of Brest* (France). During this visit, he studied jointly with Eric Loubeau and Guillaume Deschamps twistorial harmonic maps and morphisms. One of their objectives is to obtain new natural examples of harmonic maps between compact Riemannian manifolds.