David Herron: Bilipschitz homogoneity, inner diameter distance and quasiconformal and Riemann mappings.

Abstract: This is a report on work with David Freeman.

A metric space is bilipschitz homogeneous if there is a family of uniformly bilipschitz self-homeomorphisms of the space which acts transitively on it; i.e., there is a constant L such that for each pair of points x, y there exists an L-bilipschitz self-homeomorphism which maps x to y. One can view bilipschitz homogeneity as a generalization of the well-known notion of self-similarity.

Our interest here is in the study of bilipschitz homogeneous metric Jordan curves. We briefly mention the possible values of various dimensions of such curves as well as the canonical dimension gauges associated with these.

We introduce inner diameter distance Jordan disks; these are non-complete metric surfaces whose inner diameter distance completion are topologically a closed disk or a closed half-plane. These spaces have metric boundaries which are Jordan curves, and we study the consequences of their boundary being bilipschitz homogeneous. One especially important result is that such a boundary must satisfy the venerable bounded turning condition.

Roughly speaking, a simply connected plane domain (which is not the entire plane) is an inner diameter Jordan disk if and only if its boundary is locally connected. We describe when such a region has a bilipschitz homogeneous boundary in terms of quasiconformal maps, and also in terms of their Riemann maps.