

Aurelian Gheondea: Analytic Hermitian Kernels.

Abstract: The study of Kolmogorov decompositions of Hermitian kernels is presented. (1) Conditions are given for a moment representation of a given family of complex numbers in terms of linear functionals on the algebra of polynomials in a finite number of noncommuting indeterminates. (2) Conditions are derived for the existence of Stinespring dilations of Hermitian maps on a unital $*$ -algebra. (3) The boundedness conditions of Schwartz type are related to an extension of the notion of completely positive maps on a unital C^* -algebra. (4) The class of Hermitian kernels representable in the form $K(x, y) = V(x)^*UV(y)$, with U a contraction, is characterized in terms of a notion of decomposability. (5) A theorem of D. Alpay is generalized to holomorphic kernels of a possibly infinite number of variables. Let B_r be the open ball in a Hilbert space of radius $r > 0$ centered at the origin. If K is a scalar-valued Hermitian holomorphic kernel on $B_r \times B_r$, then for some $0 < r' \leq r$, the restriction of K to $B_{r'} \times B_{r'}$ admits a Kolmogorov decomposition.