Heinrich Begehr: Polyharmonic Green functions and boundary value problems.

Abstract: Iteration of harmonic Green, Neumann and Robin functions lead to different hybrid biharmonic Green functions. Adding the biharmonic Green-Almansi function and continuing the iteration leads to a variety of hybrid polyharmonic Green functions. Each of them gives rise to a particular polyharmonic boundary value problem. The process can be used for any regular domain independently of the space dimension. But in order to receive these polyharmonic fundamental solutions in explicit form particular domains have to be considered.

In the complex plane such particular domains under consideration are the unit disc, the upper half disc, circular rings, upper half circular rings, half planes, quarter planes. Even in the simplest case, the unit disc, the iteration process is not yet completed due to the complexity of the process. It is not only an analytical but also a combinatorial problem. The continued iteration of the harmonic Green function itself leads to a hierarchy of polyharmonic Green functions different from the polyharmonic Green-Almansi functions. Both types give rise to different kind of polyharmonic Dirichlet problems. Iterating these different types of polyharmonic Green functions gives again some polyharmanic Green function of higher order and another kind of related Dirichlet problem.

The harmonic Poisson kernel is related to the harmonic Green function. In fact it is the outward normal derivative of the Green function on the boundary of the domain. Some polyharmonic Poisson kernels are attained as continued primitives with respect to the Laplacian of the harmonic Poisson kernel vanishing at the boundary of the domain. They are thus solutions of some Poisson equation with homogeneous Dirichlet data on the boundary. For the disc these Poisson kernels are given in a complicated, not satisfactory form using vertical sums, see the Ph.D. thesis of J.Y. Du

http://www.diss.fu-berlin.de/diss/receive/FUDISS_thesis_000000003677.

The situation for the ring domain is more complicated and the iteration process was just started. But basic boundary value problems have been investigated; see the thesis of T. Vaitsiakhovich

http://www.diss.fu-berlin.de/diss/receive/FUDISS_thesis_000000003859.

For the upper half plane the polyharmonic Green-Almansi function is used to solve the related polyharmonic Dirichlet problem, see the thesis of. E. Gaertner

http://www.diss.fu-berlin.de/diss/receive/FUDISS_thesis_000000002129.

Some related observations for polydomains can be viewed in the Ph.D. thesis of A. Krausz

http://www.diss.fu-berlin.de/diss/receive/FUDISS_thesis_000000001659.

On the basis of some new insight in the structure of polydomains in the Ph.D. thesis of A. Mohammed

http://www.diss.fu-berlin.de/diss/receive/FUDISS_thesis_000000000885,

some fundamental boundary value problems are solved for systems of partial differential equations in several complex variables.

In the Ph.D. thesis of H. Otto

http://www.diss.fu-berlin.de/diss/receive/FUDISS_thesis_000000002246

higher-order Cauchy-Pompeiu representations in Clifford analysis are developed by the iteration procedure but boundary value problems are not studied there. Higher-order Helmholtz equations in quaternionic analysis and related Dirichlet problems are treated in the Ph.D. thesis of T.N.H.Vu

http://www.diss.fu-berlin.de/diss/receive/FUDISS_thesis_000000001591.

In the talk some basic ideas are developed for treating polyharmonic boundary value problems. In particular some new harmonic Robin functions are given for the unit disc and a circular ring. For the half disc and the half ring harmonic Dirichlet and Neumann problems are solved explicitly.