Prescribing Projections of Runge Domains in Stein Spaces *

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Dedicated to Professor Gh. Bucur 70^{th} *birthday*

1 Introduction

If $\pi : \mathbb{C}^{n+1} \to \mathbb{C}^n$ is the standard projection and D is a connected pseudoconvex domain in $\mathbb{C}^{n+1}$ then $\pi(D)$ is not necessarily pseudoconvex. Examples in this sense were given in [5] and [8]. In [3] it was proved that any connected open subset of $\mathbb{C}^n$ is the projection of a connected Runge open subset of $\mathbb{C}^{n+1}$. In this paper we will show that we can prescribe all projections of connected Runge open subsets. Moreover we will be working with products of Stein spaces. More precisely we will show that if X_1 and X_2 are Stein spaces $D_1 \subset X_1$ and $D_2 \subset X_2$ are connected open subsets then we can find D a connected Runge open subset of $X_1 \times X_2$ such that the projections of D on X_1 and X_2 are D_1 and D_2 respectively.

In [4] it was constructed a bounded and connected Runge open subset of $\mathbb{C}^n$ which has smooth boundary and whose closure is not holomorphically convex. We will show that in general the closure of a connected Runge domain in a Stein space does not enjoy any special property. That is, we will show that given a connected open set Ω in a Stein space X we can find D, a connected Runge open subset of X such that $D \subset \Omega$ and D is dense in Ω. This implies, of course, that $\overline{D} = \overline{\Omega}$.

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2 Preliminaries

Throughout this paper all Stein spaces are considered to be reduced and of finite embedding dimension.

Suppose that X is a Stein space and $\mathcal{O}$ is the sheaf of holomorphic functions on X. If D is an open subset X, D is said to be Runge in X if D is Stein and the restriction map $\mathcal{O}(X) \to \mathcal{O}(D)$ has dense image. If $K \subset X$ is a compact subset, we denote by

$$\widehat{K} = \{z \in X : |f(z)| \leq \sup_K |f| \text{ for any } f \in \mathcal{O}(X)\}$$

its holomorphically convex hull. K is called holomorphically convex (in X) if $K = \widehat{K}$. The following facts were proved in [7]:

- K is holomorphically convex if and only if it has a fundamental system of Runge neighborhoods.
- If $\phi : X \to \mathbb{R}$ is a strictly plurisubharmonic function and $c \in \mathbb{R}$ then $\{x \in X : \phi(x) < c\}$ is Runge in X. If moreover ϕ is an exhaustion then $\{x \in X : \phi(x) \leq c\}$ is compact and $\{x \in X : \phi(x) < c + \epsilon\}$ is a fundamental system of Runge neighborhoods, hence $\{x \in X : \phi(x) \leq c\}$ is holomorphically convex.

The following theorem is Theorem 5.1 in [6].

Theorem 1. *Let X be a Stein purely 1-dimensional space and $Y \subset X$ be an open subset. The following conditions are equivalent:*
i) Y is Runge in X;
ii) the natural map $H_1(Y, \mathbb{Z}) \to H_1(X, \mathbb{Z})$ is injective.

Proposition 1 was proved in [2].

Proposition 1. *Let X a Stein space $A \subset X$ be a closed analytic subset, $K \subset X$ a holomorphically convex compact subset and $L \subset A$ a holomorphically convex compact subset with $K \cap A \subset L$. Then $K \cup L$ is holomorphically convex.*

3 The Results

Proposition 2. *Suppose that X is a Stein space, K and K' are compact subsets of X such that $K \cup K'$ is holomorphically convex in X and Ω is a* *connected open subset of X such that $K \subset \Omega$ and $K' \cap \Omega = \emptyset$. Then there exists F, a connected compact subset of X, such that $K \subset F \subset \Omega$ and $F \cup K'$ is holomorphically convex.*

Remark: When $X = \mathbb{C}$, $\mathbb{C} \setminus \Omega$ is finite and $K' = \emptyset$ this result is Lemma 3 in [9].

Proof. We can assume, of course, that Ω is relatively compact in X and $K' \cap \overline{\Omega} = \emptyset$. Since $K \cup K'$ is holomorphically convex there exists a $\mathcal{C}^\infty$ strictly plurisubharmonic exhaustion function for X, $\phi : X \to \mathbb{R}$, such that $K \cup K' \subset \{x \in X : \phi(x) < 0\} \subset\subset (\Omega \cup (X \setminus \overline{\Omega}))$. It follows that $\overline{\{x \in X : \phi(x) < 0\}}$ is holomorphically convex in X. As K is compact, a finite set of connected components of $\{x \in X : \phi(x) \leq 0\}$, say $F_1, F_2 \ldots, F_p$, will cover K. Clearly $K' \cup F_1 \cup \cdots \cup F_p$ is holomorphically convex and $\cup F_j \subset \Omega$. That means that we can assume from the beginning that K has finitely many connected components. The rest of the proof will be done in several steps.

Step 1. We assume that X is purely 1-dimensional and irreducible, and $K = K_1 \cup K_2$ where K_1 and K_2 are two connected compact subsets of X. Since $K_1 \cup K_2 \cup K'$ is holomorphically convex there exists $\phi : X \to \mathbb{R}$ a $\mathcal{C}^\infty$ strictly subharmonic exhaustion function such that $K_1 \cup K_2 \cup K' \subset \{x \in X : \phi(x) < 0\} \subset\subset \Omega \cup (X \setminus \overline{\Omega})$. Moreover, since $\mathrm{Sing}(X)$ is discrete, we can assume that $\{x \in X : \phi(x) = 0\} \subset \mathrm{Reg}(X)$ and that 0 is a regular value for ϕ. It follows that $\overline{\{x \in X : \phi(x) < 0\}} = \{x \in X : \phi(x) \leq 0\}$ is holomorphically convex in X. Let D_1 and D_2 the connected components of $\{x \in X : \phi(x) < 0\}$ that contain K_1 and K_2 respectively. We have that $\overline{(D_1 \cup D_2)} = \overline{D_1} \cup \overline{D_2}$ and $\overline{D_1} \cup \overline{D_2} \cup K'$is holomorphically convex. Of course it might happen that $D_1 = D_2$ and then we can set $F = \overline{D_1}$. Suppose that $D_1 \neq D_2$. Then, as we assumed that 0 is a regular value for ϕ we have that $\overline{D_1} \cap \overline{D_2} = \emptyset$. Let $p_1 \in D_1$ and $p_2 \in D_2$ any two points. Since Ω is connected there exists $\gamma : [0, 1] \to \Omega$ a path with $\gamma(0) = p_1$ and $\gamma(1) = p_2$. On the other hand, as we assumed that X is irreducible, hence $\mathrm{Reg}(X)$ is connected, we can assume that $\gamma([0, 1]) \subset \mathrm{Reg}(X)$ and that $\gamma_{|(0,1)}$ is a $\mathcal{C}^\infty$ submersion. If we let $t_0 = \max\{t \in [0, 1] : \gamma(t) \in \overline{D_1}\}$ and $t_1 = \min\{t \in [0, 1] : \gamma(t) \in \overline{D_2}\}$ and we replace γ by $\gamma_{|[t_0,t_1]}$ we can assume on one hand that $\gamma(0, 1) \cap (\overline{D_1} \cup \overline{D_2}) = \emptyset$ and that $\overline{D_1} \cup \overline{D_2} \cup \gamma([0, 1])$ is connected. At the same time by perturbing a little bit γ we can assume that, around $t = 0$ and $t = 1$, $\gamma([0, 1])$ is orthogonal to $\{x \in X : \phi(x) = \epsilon\}$ for $\epsilon > 0$ close enough to zero. We claim that $\overline{D_1} \cup \overline{D_2} \cup \gamma([0, 1]) \cup K'$ is holomorphically convex. This is equivalent to $\overline{D_1} \cup \overline{D_2} \cup \gamma([0,1]) \cup K'$ having a fundamental system of Runge neighborhoods. Note that if we denote by $D_{1,\epsilon}$ and $D_{2,\epsilon}$ the connected components of $\{x \in X : \phi(x) < \epsilon\}$ that contain K_1 and K_2 respectively, and if we choose $D_{3,\epsilon} \subset ((X \setminus \overline{\Omega}) \cap \{x \in X : \phi(x) < 0\})$ a fundamental system of Runge neighborhoods for K' then $\{D_{1,\epsilon} \cup D_{2,\epsilon} \cup D_{3,\epsilon}\}_{\epsilon>0}$ is a fundamental system of Runge neighborhoods for $\overline{D_1} \cup \overline{D_2} \cup K'$ Since γ is a submersion, $\gamma(0,1)$ has a tubular neighborhood, U. Let $f : U \to \gamma(0,1)$ be a deformation retract. By the above orthogonality we can assume that there exists $\delta_1, \delta_2 \in (0,1)$ such that, for any small enough $\epsilon > 0$, $D_{1,\epsilon} \cup f^{-1}(\gamma(0,\delta_1))$ has a deformation retract onto $D_{1,\epsilon}$ and $D_{2,\epsilon} \cup f^{-1}(\gamma(\delta_2, 1))$ has a deformation retract onto $D_{2,\epsilon}$. At the same time $H_1(f^{-1}(\gamma(\frac{\delta_1}{2}, \delta_1)), \mathbb{Z}) = 0$, $H_1(f^{-1}(\gamma(\frac{\delta_1}{2}, \frac{1+\delta_2}{2})), \mathbb{Z}) = 0$, $H_1(f^{-1}(\gamma(\delta_2, \frac{1+\delta_2}{2})), \mathbb{Z}) = 0$. It follows from a standard Mayer-Vietoris argument that the natural map $H_1(D_{1,\epsilon} \cup D_{2,\epsilon} \cup D_{3,\epsilon}, \mathbb{Z}) \to H_1(D_{1,\epsilon} \cup D_{2,\epsilon} \cup U \cup D_{3,\epsilon}, \mathbb{Z})$ is an isomorphism. Since $D_{1,\epsilon} \cup D_{2,\epsilon} \cup D_{3,\epsilon}$ is Runge in X, Theorem 1 implies that the natural map $H_1(D_{1,\epsilon} \cup D_{2,\epsilon} \cup D_{3,\epsilon}, \mathbb{Z}) \to H_1(X, \mathbb{Z})$ is injective and therefore the natural map $H_1(D_{1,\epsilon} \cup D_{2,\epsilon} \cup U \cup D_{3,\epsilon}, \mathbb{Z}) \to H_1(X, \mathbb{Z})$ is injective. Applying again Theorem 1 we deduce that $D_{1,\epsilon} \cup D_{2,\epsilon} \cup U \cup D_{3,\epsilon}$ is Runge in X. Clearly when we shrink U and let $\epsilon \to 0$ we obtain a fundamental system of neighborhoods for $\overline{D_1} \cup \overline{D_2} \cup \gamma([0,1]) \cup K'$.

Step 2. We assume that X is purely 1-dimensional and irreducible (and no other condition on K). By the discussion that we started the proof with, we can assume that K has finitely many connected components say $K_1, K_2, \ldots, K_n$. Let Ω' be a connected open set such that $K_1 \cup K_2 \subset \Omega' \subset \Omega$ and $(K_3 \cup \cdots \cup K_n \cup K') \cap \overline{\Omega'} = \emptyset$. Using Step 1 we can find F_1 a connected compact set such that $K_1 \cup K_2 \subset F_1 \subset \Omega'$ and $F_1 \cup K_3 \cup \cdots \cup K_n \cup K'$ is holomorphically convex. We apply this argument $n-1$ times and we find F.

Step 3. We assume that X is purely 1-dimensional (not necessarily irreducible). Since we assumed that Ω is relatively compact in X, Ω intersects only finitely many irreducible components of X, say $X_1, X_2, \ldots, X_p$, and $A = \Omega \cap (\cup_{i \neq j}(X_i \cap X_j))$ is finite (we assume that $\Omega \cap X_j \neq \emptyset$ for $j = 1, 2 \ldots, p$). It follows that $K \cup K' \cup A$ is holomorphically convex. Let $\{\Omega_{j,s}\}_s$ be the finitely many connected components of $\Omega \cap X_j$ that intersect $K \cup A$. Note that, since $(K \cup A) \cap X_j \subset \cup_s \Omega_{j,s}$ and $\{\Omega_{j,s}\}_s$ are pairwise disjoint, it follows that $(K \cup A) \cap \Omega_{j,s}$ are compact. As in Step 2 there exist $F_{j,s}$ connected compact sets such that $(K \cup A) \cap \Omega_{j,s} \subset F_{j,s} \subset \Omega_{j,s}$ and $(\cup_s F_{j,s}) \cup (K' \cap X_j)$ is holomorphically convex. Applying now Proposition 1 we deduce that $(\cup_{j,s} F_{j,s}) \cup K'$ is holomorphically convex. Let $F = \cup_{j,s} F_{j,s}$

It remains to notice that F is connected. This is equivalent to showing that for any two points $x, y \in F$ there exists a connected set C such that $x, y \in C \subset F$. So let $x, y \in F$. In particular $x, y \in \Omega$ which is connected and open, hence path connected. Let γ be a path that joins x and y. If there exists j such that $\gamma \subset X_j$ then x and y are in the same connected component of $\Omega \cap X_j$. Hence there exists s such that $x, y \in \Omega_{j,s}$. On the other hand since $x, y \in F$ there exist j_1, s_1, j_2, s_2 such that $x \in F_{j_1,s_1}$ and $y \in F_{j_2,s_2}$. If $j_1 = j$ then $s_1 = s$ and therefore $x \in F_{j,s}$. If $j_1 \neq j$, as $F_{j_1,s_1} \subset X_{j_1}$ it follows that $x \in X_j \cap X_{j_1} \subset A$. We deduce that $x \in (K \cup A) \cap \Omega_{j,s}$ which is contained in $F_{j,s}$. In both cases we get that $x \in F_{j,s}$. Similarly $y \in F_{j,s}$ and $F_{j,s}$ is connected. If there is no j such that $\gamma \subset X_j$ we can write γ as $\gamma = \gamma_1\gamma_2 \cdots \gamma_q$ where for each $l = 1, \ldots, q$ there exists j_l such that $\gamma_l \subset X_{j_l}$ and $j_l \neq j_{l+1}$. Let a_{l-1} and a_l be the endpoints of γ_l, $a_0 = x$, $a_q = y$. However from $a_l \in X_{j_l} \cap X_{j_{l+1}}$ we get that $a_l \in A$ and therefore $a_l \in F$. By what we said before we can find $C_1, \ldots, C_q$ connected sets such that $a_{l-1}, a_l \in C_l \subset F$. Then $x, y \in \cup C_l \subset F$ and $\cup C_l$ is connected.

Step 4. The general case. As we mentioned at the beginning we can assume that K has finitely many connected components. As in Step 2, it suffices to prove the proposition when K has two connected components, K_1 and K_2. For every $x_0 \in \Omega$ let $\Omega(x_0)$ be the set of all $x \in \Omega$ such there exist $x_1, x_2, \ldots, x_n = x$ points in Ω, there exist $Y_1, \ldots Y_n$ purely 1-dimensional Stein subspaces of X and there exist $U_1, \ldots, U_n$ such that each U_j is a connected open subset of $Y_j \cap \Omega$ (open in the topology of X_j) and $U_j \supset \{x_{j-1}, x_j\}$. It follows from the arguments in [1] that $\Omega(x_0)$ is open for every $x_0 \in \Omega$. Since $\Omega(x)$ and $\Omega(y)$ are either disjoint or coincide, $\cup_{x \in \Omega} \Omega(x) = \Omega$ and Ω is connected it follows that $\Omega(x) = \Omega$ for every x. We choose two points $x_0 \in K_1$ and $x \in K_2$. We have that $x \in \Omega(x_0)$ and therefore there exist $x_1, x_2, \ldots, x_n = x$, $Y_1, \ldots Y_n$ and $U_1, \ldots, U_n$ as above. We can assume that for $i \neq j$ Y_i and Y_j have no common irreducible component and, if we shrink U_j we can assume that $U_j \cap Y_{j+1} = \{x_j\}$, $U_j \cap Y_{j-1} = \{x_{j-1}\}$ and $U_j \cap Y_i = \emptyset$ for $i \notin \{j-1, j, j+1\}$. It follows that $U = \cup U_j$ is a connected open subset of the 1-dimensional Stein space $Y = \cup Y_j$. Let $V_1, \ldots, V_r$, $V_1 \supset U$, the connected components of $\Omega \cap Y$ that intersect K. Since $K \cap Y \subset \cup V_j$ it follows that each $K \cap V_j$ is compact and $(K \cap V_1) \cup \cdots \cup (K \cap V_r) \cup (K' \cap Y)$ is holomorphically convex. By Step 3 we choose L a connected compact set such that $(K \cap V_1) \subset L \subset V_1$ and $L \cup (K \cap V_2)) \cup \cdots \cup (K \cap V_r) \cup K' \cap Y$ is holomorphically convex. By Proposition 1 we have that $[K \cup K'] \cup [L \cup (K \cap V_2)) \cup \cdots \cup (K \cap V_r) \cup (K' \cap Y)]$ is holomorphically convex. However $[K\cup K']\cup[L\cup(K\cap V_2))\cup\cdots\cup(K\cap V_r)\cup(K'\cap Y)] = K\cup L\cup K'$ and $K\cup L = K_1\cup K_2\cup L$. As K_1, K_2 and L are connected and $K_1\cap L\neq\emptyset$, $K_2\cap L\neq\emptyset$ we get that $F := K\cup L$ is connected.

□

Theorem 2. *Suppose that X_1 and X_2 are two Stein spaces, D_1 is a connected open subset of X_1 and D_2 is a connected open subset of X_2. Let $\pi_1 : X_1\times X_2\to X_1$ be the projection on the first coordinate and $\pi_2 : X_1\times X_2\to X_2$ be the projection on the second coordinate. Then there exists D a connected open subset of $X_1\times X_2$ such that D is Runge in $X_1\times X_2$, $\pi_1(D) = D_2$ and $\pi_2(D) = D_2$.*

Proof. Let $\{P_k\}_{k\geq 1}$ be a sequence of holomorphically convex compact subsets of X_1 and $\{Q_k\}_{k\geq 1}$ be a sequence of holomorphically convex compact subsets of X_2 such that $\cup P_k = D_1$ and $\cup Q_k = D_2$. We will construct inductively a sequence $\{V_m\}_{m\geq 1}$ of connected open subsets of $X_1\times X_2$ and $\{F_m\}_{m\geq 1}$ a sequence of compact subsets of $X_1\times X_2$ with the following properties:
1) $V_m\subset F_m\subset V_{m+1}$ and $\pi_j(F_m)\subset D_j$ for all $m\geq 1$ and $j\in\{1,2\}$
2) V_m is Runge in $X_1\times X_2$ and F_m is holomorphically convex in $X_1\times X_2$ for all $m\geq 1$
3) $\pi_1(V_{2m-1})\supset P_m$ and $\pi_2(V_{2m})\supset Q_m$

Note that if we manage to construct $\{V_m\}_{m\geq 1}$ with these three properties and we set $D = \cup V_m$ then since $\{V_m\}_{m\geq 1}$ is an increasing sequence of connected Runge open subsets of $X_1\times X_2$ it follows that D is connected and Runge and properties 1) and 3) guarantee that $\pi_1(D) = D_2$ and $\pi_2(D) = D_2$.

Let x_1 be any point in X_2. It follows that $P_1\times\{x_1\}$ is holomorphically convex in $X_1\times X_2$. We can find then $\phi : X_1\times X_2\to\mathbb{R}$ a $\mathcal{C}^\infty$ strictly plurisubharmonic exhaustion for $X_1\times X_2$ such that $P_1\times\{x_1\}\subset\{x\in X_1\times X_2 : \phi(x)<0\}\subset\subset D_1\times D_2$. We choose V_1 to be the connected component of $\{x\in X_1\times X_2 : \phi(x)<0\}$ that contains $P_1\times\{x_1\}$ and $F_1 = \{x\in X_1\times X_2 : \phi(x)\leq 0\}$.

Suppose now that we have constructed $V_1,\ldots,V_{2m-1}$ and $F_1,\ldots,F_{2m-1}$ and we construct V_{2m} and F_{2m}. Since $\pi_1(F_{2m-1})$ is a compact subset of D_1, $\pi_1(F_{2m-1})\neq D_1$. Let $x_{2m}\in D_1\setminus\pi_1(F_{2m-1})$ be any point. We have that $\{x_{2m}\}\times Q_{2m}$ is holomorphically convex. Moreover since $\{x_{2m}\}\times X_2$ is an analytic subset of $X_1\times X_2$ and $(\{x_{2m}\}\times X_2)\cap F_{2m-1} = \emptyset$ it follows that $(\{x_{2m}\}\times Q_{2m})\cup F_{2m-1}$ is holomorphically convex. By Proposition 2 there exists F a connected holomorphically convex compact subset of X such that $(\{x_{2m}\}\times Q_{2m})\cup F_{2m-1}\subset F\subset D_1\times D_2$. Let $\phi : X_1\times X_2\to\mathbb{R}$

be a $\mathcal{C}^\infty$ strictly plurisubharmonic exhaustion for $X_1 \times X_2$ such that $F \subset \{x \in X_1 \times X_2 : \phi(x) < 0\} \subset\subset D_1 \times D_2$. We choose V_{2m} to be to be the connected component of $\{x \in X_1 \times X_2 : \phi(x) < 0\}$ that contains F and $F_{2m} = \{x \in X_1 \times X_2 : \phi(x) \leq 0\}$. The construction of V_{2m+1} and F_{2m+1} once that we have constructed $V_1, \ldots, V_{2m}$ is completely similar. □

Proposition 3. *Suppose that X is Stein space and Ω is a connected open subset of X. Then there exists D a connected open subset of Ω such that D is Runge in X and D is dense in Ω.*

Proof. Let $\{x_m\}_{m\geq 1}$ be a countable dense subset of Ω. As in the proof of Theorem 2 we will construct inductively a sequence $\{V_m\}_{m\geq 1}$ of connected open subsets of Ω and $\{F_m\}$ a sequence of compact subsets of Ω with the following properties:
1) $V_m \subset F_m \subset V_{m+1}$ and $x_m \in V_m$ for all $m \geq 1$ and $j \in \{1, 2\}$
2) V_m is Runge in Ω and F_m is holomorphically convex in Ω for all $m \geq 1$.

Once that we have constructed this sequences we set $D = \cup V_m$. The construction of V_1 and F_1 is straightforward. We assume that we have constructed $V_1, \ldots, V_{m-1}$ and $F_1, \ldots, F_{m-1}$ and we construct V_m and F_m. Since F_{m-1} is holomorphically convex, $F_{m-1} \cup \{x_m\}$ is also holomorphically convex. By Proposition 2 we can find a connected holomorphically convex compact subset of X such that $F_{m-1} \cup \{x_m\} \subset F \subset \Omega$. Let $\phi : X \to \mathbb{R}$ be a $\mathcal{C}^\infty$ strictly plurisubharmonic exhaustion for X such that $F \subset \{x \in X : \phi(x) < 0\} \subset\subset \Omega$. We choose V_m to be the connected component of $\{x \in X : \phi(x) < 0\}$ that contains F and $F_m = \{x \in X : \phi(x) \leq 0\}$.

□

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