Several Complex Variables

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Articles:

- **Mihnea Colţoiu**, **Cezar Joiţa**: *On the separation of the cohomology of universal coverings of 1-convex surfaces*, Advances in Mathematics **265** (2014), 362–370.
- **George-Ionuţ Ioniţă**: *q-Completeness of unbranched Riemann domains over complex spaces with isolated singularities* to appear in Complex Variables and Elliptic Equations.
- **Ovidiu Preda**: *Locally Stein Open Subsets in Normal Stein Spaces*, to appear in Journal of Geometric Analysis
- **Eugen Mihăilescu**, *Geometric and ergodic aspects in conformal dynamics on invariant fractals*, Annals of the University of Bucharest (mathematical series) **5 (LXIII)** (2014), 157–168.

Brief description of our results:

Mihnea Colţoiu, Cezar Joiţa: On the separation of the cohomology of universal coverings of 1-convex surfaces.

If X is a 1-convex complex space, that is a proper modification of a Stein space at a finite set, then for every coherent analytic sheaf $\mathcal{F}$ on X the cohomology groups $H^q(X, \mathcal{F})$ are finite dimensional for every $q \geq 1$. In particular they are separated.

If $p : \tilde{X} \to X$ is the universal covering of X, the only geometric property of $\tilde{X}$ is that it is p_3-convex in the sense of Grauert and Docquier (M. Colţoiu, Comment. Math. Helv. (1993)). A natural question in this context is to decide if $\tilde{X}$ has separated cohomology for the structure sheaf.

The main result of this paper is the following theorem:

Theorem. *There exists a 1-convex complex surface X such that its universal covering $\tilde{X}$ has the property that $H^1(\tilde{X}, \mathcal{O}_{\tilde{X}})$ is not separated.*

Our main ingredients for the proof are the following:

- the construction (M. Colţoiu , C. Joiţa, Mathematische Zeitschrft 2013) of a 1-convex surface X such that its universal covering does not satisfy the discrete Kontinuitätssatz,
- the p_3-convexity of $\tilde{X}$,
- the Serre duality.

George-Ionuţ Ioniţă: q-Completeness of unbranched Riemann domains over complex spaces with isolated singularities

The Levi problem on complex spaces asks to decide if a locally Stein open subset in a Stein complex space is Stein. In Mathematische Annalen in 2007, M. Colţoiu and K. Diederich proved that if $p : Y \to X$ is an unbranched Riemann domain, X and Y are complex spaces with isolated singularities and p p is a Stein morphism, then Y is Stein. The goal of this article is to extend Colţoiu and Diederich's result for q-complete spaces. The following theorem is proved:

Theorem. *Let X and Y be complex spaces with isolated singularities and $p : Y \to X$ an unbranched Riemann domain. Assume that X is a q-complete space and that p is a Stein morphism, i.e. each point $x \in X$ has a neighborhood $V = V(x)$ such that $p^1(V)$ is Stein. Then Y is also q-complete*

If X si Y are smooth, the result was obtained by V. Vâjâitu in Ann. Sc. Norm. Sup. Pisa in 2000.

Ovidiu Preda: Locally Stein Open Subsets in Normal Stein Spaces

The main result of this paper is the following theorem:

Theorem. *Let X be a Stein normal complex space, Ω a locally Stein open subset of X and Y the singular locus of X. Then for every sequence of points $(x_n)_n$ in Ω which tends to a limit $x \in \partial\Omega \setminus Y$ there exists a holomorphic function on Ω which is unbounded on $(x_n)_n$.*

For a bounded domain Ω the result was obtained by J.E. Fornaess and R. Narasimhan.

As an application of this theorem, it is proved that a holomorphic locally trivial fiber bundle with Stein base and bounded domain of holomorphy in $\mathbb{C}^n$ as fiber is Stein if and only if it has an envelope of holomorphy.

Eugen Mihăilescu, Geometric and ergodic aspects in conformal dynamics on invariant fractals

We survey recently discovered aspects in the geometric theory of higher dimensional dynamical systems on basic fractal sets. For most of these systems, hyperbolicity and the various invariant measures play a central role. We present several results which underline the rich connections between geometric theory of currents, ergodic theory, and thermodynamical formalism. Also we explain the interplay between the geometry of folded fractal sets and the ergodic properties of equilibrium measures supported on them.