Several Complex Variables

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Articles:

• **M. Colţoiu**, K. Diederich, **C. Joiţa**: *On complex spaces with prescribed singularities*, acceptat la Mathematical Research Letters

• **E. Mihăilescu**, B. Stratmann: *Upper estimates for stable dimension of fractal sets with variable number of foldings*, acceptat la International Mathematics Research Notices.

• **M. Colţoiu**, **C. Joiţa**: *Convexity properties of coverings of 1-convex surfaces*, Mathematische Zeitschrift, 275 (2013), 781–792.

• **I. Chiose**: *Obstructions to the existence of Kähler structures on compact complex manifolds*, acceptat la Proceedings of the AMS.

• **E. Mihăilescu**, M. Urbanski: *Hausdorff dimension of limit sets of countable conformal iterated function systems with overlaps*, Contemporary Mathematics, vol. 600.

Brief description of our results:

M. Colţoiu, K. Diederich, C. Joiţa: On complex spaces with prescribed singularities.

We prove the following theorem:

Theorem. *Let Y be a reduced complex space. Then there exists a reduced complex space X such that:*
1) $Sing(X) = Y$, $\dim(X) = \dim(Y) + 2$.
2) along $Reg(Y)$, the complex space X has only quadratic singularities, (i.e. the product of a complex manifold of dimension $n = \dim(Y)$ and a surface with an isolated quadratic 2-dimensional singularity).

Moreover, if Y is normal then X can be chosen to be normal and if Y is locally irreducible then X can be chosen to be locally irreducible.

For the proof we consider a resolution of singularities $\pi : \tilde{Y} \to Y$ and over $\tilde{Y}$ we consider a rank 2 vector bundle $E \to \tilde{Y}$ which is relatively negative. On each fiber of E we have the equivalence relation $x \sim (-x)$. If we let $F := E/\sim$ we obtain a locally trivial fibration $\tau : F \to \tilde{Y}$ with typical fiber $\{(z_1, z_2, z_3) \in \mathbb{C}^3 : z_1 z_2 = z_3^2\}$ which has a quadratic 2-dimensional isolated singularity. From F we get the desired complex space X by applying the relative Remmert quotient theorem and Wiegmann quotient theorem.

In the embedded case, i.e. if Y is a complex subspace of a complex manifold Z, we give another construction of X using only Wiegmann quotient theorem. In this particular case we obtain:

Theorem. *Suppose that Z is a complex manifold and Y is a closed subspace of Z. Then there exists a complex space X with the following properties:*
1) Sing$(X) = Y$ and $\dim(X) = \dim(Z) + 1$.
2) X is locally irreducible.
3) The normalization of X is smooth and therefore X is not normal at any point of Y.
4) If Z is connected then X is irreducible.

E. Mihăilescu, B. Stratmann: Upper estimates for stable dimension of fractal sets with variable number of foldings.

For a hyperbolic map f on a saddle-type fractal Λ with self-intersections, the number of f-preimages of a point $x \in \Lambda$ may depend on x. This makes estimates of the stable dimensions more difficult than for diffeomorphisms or for maps which are constant-toone. We employ the thermodynamic formalism in order to derive estimates for the stable Hausdorff dimension function δ^s on Λ, in the case when f is conformal on local stable manifolds. The following theorem is proved:

Theorem. *Let $f : M \to M$ be a $\mathcal{C}^2$-endomorphism which is c-hyperbolic on a basic set Λ of f and for which there exists a continuous function $\omega : \Lambda \to (0, \infty)$ such that $\Delta(x) \geq \omega(x)$, for all $x \in \Lambda$. Then, the following upper estimate is true for any point $x \in \Lambda$:*

$$\delta^s(x) \leq t_\omega,$$

where t_ω is the unique zero of the pressure function $t \to P(t\Phi^s - \log \omega)$, associated to the potential function $t\Phi^s - \log \omega$ pe Λ on Λ.

Here Δ is the preimage counting function of in Λ and Φ^s is the stable potential function.

M. Colţoiu, C. Joiţa: Convexity properties of coverings of 1-convex surfaces

The well-known Shafarevich Conjecture asserts that the universal covering space of a projective algebraic manifold is holomorphically convex. Although there are partial results, a complete answer to this problem is not known even for surfaces.

In this paper we are interested in studying convexity properties of the universal covering of 1-convex surfaces. We recall that projective algebraic manifolds are a particular case of Moishezon manifolds, that the exceptional set of a 1-convex manifold is a Moishezon space and that every Moishezon space is the exceptional set of a 1-convex space.

If X is a 1-convex surface and $p : \tilde{X} \to X$ is a covering map, it was proved by M. Colţoiu that in general $\tilde{X}$ is not holomorphically convex. In fact $\tilde{X}$ might not be even weakly 1-complete (that is, $\tilde{X}$ might not carry a continuous plurisubharmonic exhaustion function). However $\tilde{X}$ can be exhausted by a sequence of strongly pseudoconvex domains and therefore $\tilde{X}$ satisfies the continuous disk property. We investigate the *discrete* disk property for $\tilde{X}$ which definitely is a stronger property. We prove the following:

Theorem. *There exists a 1-convex surface whose universal covering does not satisfy the discrete disk property.*

We remark that our example must contain an infinite Nori string of rational curves because Colţoiu and Joiţa proved that if $\tilde{X}$ does not contain an infinite Nori string of rational curves then actually $\tilde{X}$ does satisfy the discrete disk property.

I. Chiose: Obstructions to the existence of Kähler structures on compact complex manifolds

A compact complex manifold X is in class $\mathcal{C}$ if there exists a complex Kähler space Y and a surjective meromorphic map $h : Y \to X$. We prove the following theorem:

Theorem. *Let X be a compact complex manifold of dimension n in the Fujiki class $\mathcal{C}$ and suppose there exists ω a strictly positive (1,1) form on X such that $i\partial\overline{\partial}\omega = 0$. Then X is a Kähler manifold.*

This theorem implies immediately:

Theorem. *Let X be a compact complex manifold of dimension n in the Fujiki class $\mathcal{C}$ and which is not Kähler. Then there exists a positive, nonzero current T of bidegree $(n-1, n-1)$, which is $i\partial\overline{\partial}$-exact.*

The two theorems are generalizations to the analytic case of the algebraic case which was proved by Peternell.

E. Mihăilescu, M. Urbanski: Hausdorff dimension of limit sets of countable conformal iterated function systems with overlaps.

In this paper we provide lower and upper estimates for the Hausdorff dimension of the limit sets of conformal iterated function systems with overlaps. What is most important is that the alphabet of those system, though countable, is allowed to be infinite. the case of finite alphabet was studied also by Mihăilescu and Urbanski. As in that case, these estimates are expressed in terms of the topological pressure and the function $d(\cdot)$ counting overlaps. However, the infinite case introduces new difficulties. In the case when the function $d(\cdot)$ is constant, we get an exact formula for the Hausdorff dimension. We also prove that in certain cases this formula holds if and only if the function $d(\cdot)$ is constant. In the end, we also give examples of countable IFS with overlaps.