Scientific Report
project PN-II-ID-PCE-2011-3-0045
November 2011 – October 2016

1 General presentation of the results and scientific activity

• The members of the research team of this project [Lucian Beznea, Oana Lupaşcu, Gabriela Marinoschi, and Ioan R. Ionescu (Project Coordinator)] started the activity in this project in November 2011 and they worked continuously during the years 2012, 2013, 2014, 2015, and 2016.

A main step of the research was to find a mathematical framework (the equations for the shallow flow of a visco-plastic fluid/solid) and a numerical one, convenient to the avalanches modeling. These equations lead to a limit analysis problem for the ductile rupture which modelises the onset of the dense avalanches. The approach for the limit analysis problem was the Discontinuous Velocity Domain Splitting method (abbreviated DVDS method).

In addition to these research themes, there were obtained preparatory results, in preparation of a stochastic model of the fragmentation phase of an avalanche. First, it was proved an existence result for right Markov processes having càdlàg trajectories. Then, this method was applied for both discrete and continuous branching processes. Branching Markov processes on the finite configurations of a given base space have been constructed and studied. There were established branching properties for the solution of a stochastic differential equation of fragmentation for mass distribution and we succeeded to associate a càdlàg Markov process on the space of all fragmentation sizes, introduced by J. Bertoin. It was constructed a measure-valued branching Markov process associated with a nonlinear Neumann boundary value problem, where the boundary condition has a nonlinear pseudo monotone branching mechanism term. It was shown an equivalence between the Dirichlet and the Neumann problem for the Laplace operator, on finite and infinite dimensional balls. Also, it was shown that the subordination induced by a convolution semigroup (subordination in the sense of Bochner) of a C_0-semigroup of sub-Markovian operators on an L^p space is actually associated to a subordination of a right (Markov) process.

Another objective was the study of the diffusion processes in inhomogeneous media, respectively the optimization of the diffusive flow with a transport term, acting with a distributed control and by the identification of a source in a diffusion process, based on some observations on the solution. Both problems focus on the study of the singular degenerate diffusion equations. It was investigated a non-convex optimal control problem, governed by a nonlinear elliptic equation with a discontinuous diffusion term.

Further, it was studied a reaction-diffusion model with cross diffusion, which describes the directed motion of a population of bacteria b to a chemoattractant signal c, in a stratified, inhomogeneous media, with n layers. The study of this model is motivated by a bioremediation problem of a porous medium (soil) polluted by a substance c. It was

studied the dynamics of a fish population with age-structure and space diffusion, under a renewal condition represented by various non-local nonlinear stock- recruitment functions.

- There are 22 scientific works (published or accepted for publication) elaborated in the frame of the project.
- Results obtained in this project were presented in 62 talks, delivered in international conferences, symposia, and scientific seminars.
- Four research stages and one scientific visit of O. Lupascu (PhD student at IMAR) were supported from the project. She also participated in three international conferences and two summer schools. The members of the research team of the project co-organized four international meetings (workshops and conferences).
- In December 2014 O. Lupascu successfully defended her PhD thesis, entitled "Probabilistic and deterministic models for fracture type phenomena", with the topic in the scientific area of the project.

2 Scientific activities resulting from the project

2.1 Articles published or accepted for publication, that mention the project

[1] Ioan R. Ionescu, Viscoplastic shallow flow equations with topography, *Journal of Non-Newtonian Fluid Mechanics* **193** (2013), 116–128. (2015 IF: 2.172)

[2] Ioan R. Ionescu, Augmented Lagrangian for shallow viscoplastic flow with topography, *Journal of Computational Physics* **242** (2013), 544–560. (2015 IF: 2,556)

[3] Miruna Beldiman, Ioana Boaca, Gabriela Marinoschi, Optimization of a singular flow in a porous medium, *ZAMM-Journal of Applied Mathematics and Mechanics* **93** (2013), 633–647. (2015 IF: 1,293)

[4] Lucian Beznea, Oana Lupaşcu, Andrei-George Oprina, A unifying construction for measure-valued continuous and discrete branching processes. In: *Complex Analysis and Potential Theory, CRM Proceedings and Lecture Notes*, vol. **55**, Amer. Math. Soc., Providence, RI, 2012, 47–59.

[5] Angelo Favini, Alberto Lorenzi, Gabriela Marinoschi, Hiroki Tanabe, Perturbation methods and identifcation problems for degenerate evolution systems, In: *Advances in Mathematics, Invited Contributions at the Seventh Congress of Romanian Mathematicians*, Brasov, 2011 (Eds. L. Beznea, V. Brinzanescu, M. Iosifescu, G. Marinoschi, R. Purice, D. Timotin), Publishing House of the Romanian Academy, 145–156, 2013.

[6] Oana Lupaşcu, Subordination in the sense of Bochner of L^p-semigroups and associated Markov processes. *Acta Mathematica Sinica, English Series* **30** (2014), 187–196. (2015 IF: 0,386)

[7] Gabriela Marinoschi, A control problem for a cross-diffusion system in a non-homogeneous medium, *Journal Biological Dynamics* **7** (Suppl. 1) (2013), 88–107.

[8] L. Beznea, A.-G. Oprina: Bounded and L^p-weak solutions for nonlinear equations of measure-valued branching processes, *Nonlinear Analysis, Th., Methods & Appl.* **107** (2014), 34–46. (2015 IF: 1.125)

[9] L. Beznea, M. Rockner: On the existence of the dual right Markov process and applications, *Potential Analysis*, 2014, DOI 10.1007/s11118-014-9447-0. (2015 IF: 0.956)

[10] G. Marinoschi, Control approach to an ill-posed variational inequality, *Math. Model. Nat. Phenom.* **9** (2014), 153–170. (2015 IF: 0,725)

[11] G. Fragnelli, G. Marinoschi, R.M. Mininni, S. Romanelli, A control approach for an identification problem associated to a strongly degenerate parabolic system with interior degeneracy, in: New Prospects in *Direct, Inverse and Control Problems for Evolution Equations*, Eds. A. Favini et al., Springer INDAM Series, vol. **10**, 2014. ISBN 978-3-319-11405-7

[12] I. R. Ionescu and O. Lupaşcu, Modeling shallow avalanche onset over complex basal topography, *Advances in Computational Math*, **42** (2016), 5–26, (2015 IF: 1, 325).

[13] L. Beznea and O. Lupaşcu, Measure-valued discrete branching Markov processes, *Trans. Amer. Math. Soc.* **368** (2016), 5153–5176. (2015 IF: 1,196).

[14] L. Beznea, M. Deaconu, and O. Lupaşcu: Branching processes for the fragmentation equation, *Stochastic Processes and their Applications* **125** (2015), 1861–1885 (2015 IF: 1,193).

[15] P. Colli, G. Gilardi, G. Marinoschi, A boundary control problem for a possibly singular phase field system with dynamic boundary conditions, *Journal of Mathematical Analysis and Applications*, **434** (2016), 432–463, (2015 IF: 1,014).

[16] I. R. Ionescu, O. Lupaşcu, Onset of a dense avalanche on a plane slope, *Proc. Romanian Academy-Series A*, 16, 405–412 (2015). (2015 IF: 1,16).

[17] I. R. Ionescu, A. Mangeney, F. Bouchut, and O. Roche, Viscoplastic modelling of granular column collapse with pressure dependent rheology, *J. Non-Newtonian Fluid Mechanics* , 219 (2015) 1–18 (2015 IF: 2.172)

[18] L. Beznea, M.N. Pascu, and N. R. Pascu, An equivalence between the Dirichlet and the Neumann problem for the Laplace operator, *Potential Analysis*, **44** (2016), 655–672 (2015 IF: 0.956).

[19] L. Beznea, M. Deaconu, and O. Lupascu, Stochastic equation of fragmentation and branching processes related to avalanches, *Journal of Statistical Physics* **162** (2016), 824–841 (2015 IF: 1.537)

[21] G. Marinoschi, A singular nonconvex optimal control problem, *Pure Applied Functional Analysis* (2016), accepted

[22] L. Beznea, V. Barbu, Measure-valued branching processes associated with Neumann nonlinear semiflows, *Journal of Mathematical Analysis and Applications*, **441** (2016), 167-182 (2015 IF: 1.014)

2.2 Talks in the international conferences, symposia and seminars.

1. I. R. Ionescu, Plasticity of shallow flows with topography. Applications to snow avalanches, *Int. Plasticity Symposium*, ianuarie, 2012, San Juan, SUA, keynote speaker

2. O. Lupaşcu, Modelisation de la rupture ductile des materiaux hetereogenes avec DVDS, *Journée des doctorands*, ianuarie 2012, Universitatea Paris 13, Franta

3. I. R. Ionescu, Dynamic plasticity of crystals, *seminar IMAR*, Bucharest, March 2012

4. O. Lupaşcu, Subordination of L^p-semigroups and right processes, *Spring School in Probability*, Dubrovnik, Croatia, April 2012

5. L. Beznea, The semigroup approach for measure-valued branching processes and a nonlinear Dirichlet problem, *Seminaire Institut Elie Cartan*, Univ. de Lorraine, France, April 2012

6. O. Lupaşcu, Subordination in the sense of Bochner of L^p-semigroups and associated Markov processes *International Conference on Probability and Related Aspects*, Alba Iulia, May 2012

7. I.R. Ionescu, Modeling ductile fracture with DVDS, *Conference on Constructive Nonsmooth Analysis*, Saint-Petersburg, Russia, June 2012

8. L. Beznea, Potential theory of infinite dimensional Lévy processes, *International Conference on Controlled Deterministic and Stochastic Systems*, Iaşi, Romania, July 2012, invited speaker

9. O. Lupaşcu, Markov processes and martingale problem associated with subordinations in the sense of Bochner of L^p- semigroups, *International Conference on Controlled Deterministic and Stochastic Systems*, Iaşi, Romania, July 2012

10. L. Beznea, Non-linear PDEs and measure-valued branching Markov processess, *Seminaire Basque Center for Applied Mathematics*, Bilbao, Spain, July 2012

11. O. Lupaşcu, Markov processes and martingale problem associated with subordinations in the sense of Bochner of L^p- semigroups", *Seminarul AG Stochastik and Analysis*, Universitatea Tehnica Dresda, Germany, July 2012

12. L. Beznea, Nonlinear equations and Markov processes on the space of finite configurations, *PDEs and Stochastic Processes–The 5th Workshop Series on Mathematics*, Piteşti, October 2012

13. I. R. Ionescu, From Cheeger problem to limit analysis, *International Conference on PDEs and Stochastic Processes–The 5th Workshop Series on Mathematics*, Piteşti, October 2012

14. G. Marinoschi, Global existence for a cross-diffusion model in a stratified medium, *International Conference on PDEs and Stochastic Processes–The 5th Workshop Series on Mathematics*, Piteşti, October 2012

15. O. Lupaşcu, Markov processes and martingale problem associated with subordinations in the sense of Bochner of L^p- semigroup, *International Conference on PDEs and Stochastic Processes–The 5th Workshop Series on Mathematics*, Piteşti, October 2012

16. I. R. Ionescu, Ductile Rupture with DVDS, *World Congress on Engineering and Technology*, Beijing, China, October 2012

17. I. R. Ionescu, Dynamic plasticity of crystals, *Seminar Beihang University*, Beijing, China, November 2012

18. I. R. Ionescu, Dynamic visco-plastic crystals: an eulerian modeling," International Plasticity Symposium", Nassau, January 2013 (invited)

19. I. R. Ionescu, From Cheeger problem to limit analysis, "American Joint Mathematics Meetings", San Diego, (USA), January 2013

20. L. Beznea, invited talk at The Octav Mayer Institute of Mathematics of the Romanian Academy, Iaşi, Romania, March 2013

21. I. R. Ionescu, Shallow viscoplastic flow over a natural topography, "International Conference on Approximation Methods and Numerical Modelling in Environment and Natural Resources (MAMERN)", Granada (Spain), April 2013

22. I. R. Ionescu, Viscoplastic shallow flow equations with topography, "6e Biennale Francaise des Mathématiques Appliqueés et Industrielles", Seignosse-Le-Penon (France), May 2013

23. L. Beznea, "Joint International Meeting of the Amer. Math. Soc. and Romanian Math. Soc.", June 2013, Alba Iulia, Romania

24. I. R. Ionescu, Augmented Lagrangian for shallow viscoplastic flow with topography, "Joint International Meeting of the American Mathematical Society and the Romanian Mathematical Society", Alba-Iulia, June 2013

25. O. Lupaşcu, Mathematical modeling of the dense avalanche onset, "Joint International Meeting of the Amer. Math. Soc. and Romanian Math. Soc.", June 2013, Alba Iulia, Romania

26. L. Beznea invited talk at "Bielefeld Stochastic Summer", Bielefeld, Germania, August 2013.

27. I. R. Ionescu, Schwarz method for slip weakening friction with applications to earthquake source dynamics, "22nd International Conference on Domain Decomposition Methods", Lugano (Switzerland), September 2013

28. O. Lupaşcu, talk in the Potential Theory Seminar, organized by IMAR and University of Bucharest, Faculty of Mathematics and Computer Science, September 2013.

29. I. R. Ionescu, Modeling granular collapse with pressure dependent viscoplastic fluids, "Viscoplastic Fluids: From Theory to Applications", Rueil-Malmaison (France), November 2013

30. L. Beznea, Modification des fonctions de transitions par des noyaux et applications, Universitatea din Tunis (Institut préparatoire aux études d'ingénieurs de Tunis), Tunisa, April, 2014.

31. L. Beznea, Negative definite functions and nonlinear PDEs of measure-valued branching processes, Chalmers University and University of Göteborg, Sweden, *Analyssem-inarium*, May 2014.

32. L. Beznea, Branching processes and stochastic fragmentation equation, "7th International Conference on Stochastic Analysis and its Applications" (satellite conference of ICM 2014), Seoul National University, South Korea.

33. L. Beznea, Markov processes on the Lipschitz boundary for the Neumann and Robin problems., "Oberwolfach Workshop: Dirichlet Form Theory and Its Applications", October 20, 2014.

34. G. Marinoschi, The sand-pile model revisited, 12e Colloque Franco-Roumain de Mathématiques Appliquées, section Mecanica Fluidelor, Lyon, France, 24-30.08.2014

35. I. R. Ionescu, Anisotropic damage under dynamic loadings, " International Plasticity Symposium" , Freeport, January 2014

36. I. R. Ionescu, Numerical Modeling of Dynamic Anisotropic Damage, ICCM2014, Cambridge, UK, July 2014

37. O. Lupaşcu, Measure-valued discrete branching Markov processes and applications, Colloque Franco-Roumain de Mathematiques Appliquees, Lyon, France, August 2014

38. O. Lupaşcu, Measure-valued discrete branching Markov processes and applications (Poster), Journees Modelisation Aleatoire Statistique, Toulouse, France, August 2014

39. O. Lupaşcu, Measure-valued discrete branching Markov processes and applications, Journees de Probabilites, Marseille, France, May 2014

40. O. Lupaşcu, invited talk: Branching processes for the fragmentation equation: applications to avalanches, International Conference Avalanches and Rupture Phenomena, Nancy, Franţa, February 2015.
http://iecl.univ-lorraine.fr/%7EMadalina.Deaconu/workshop2015/index.html

41. O. Lupaşcu, invited talk: Processus de branchement pour l'équation de fragmentation. Application aux avalanches", University of Dijon, Séminaire de l'équipe de Statistique, Probabilités, Optimisation et Contrôle, Franţa, March 2015.

42. O. Lupaşcu, invited talk: Processus de branchement pour l'équation de fragmentation. Application aux avalanches, Le Congres de la Societe de Mathematiques Appliquees et Industrielles (SMAI), Les Karellis, Franţa, June 2015,
http://smai.emath.fr/smai2015/programme_detaille.php

43. O. Lupaşcu, invited talk: Branching processes and the fragmentation equation, in the section "Probability, Stochastic Analysis, and Mathematical Statistics", The Eighth Congress of Romanian Mathematicians, Iaşi, July 2015,
http://www.math.uaic.ro/cmr2015/index.php?talks

44. O. Lupaşcu, Modelarea declanşării şi a fazei de fragmentare pentru o avalanşă de zăpadă, The 36-th Conference Caius Iacob of Fluid Mechanics and its Technical Applications, Bucureşti, October 2015,
http://www.incas.ro/images/stories/Caius_Iacob_2015/book_abs_final.pdf

45. O. Lupaşcu, invited talk: Branching processes for continuous and discontinuous fragmentation kernels; application to avalanches, Colloque Franco-Roumain en Théorie des Probabilités, Bucureşti, November 2015,
http://imar.ro/organization/activities/standalone/Afis-AUF-conferinta-probab-oct15.pdf

46. L. Beznea, invited talk: The Neumann problem: an equivalence with the Dirichlet problem and associated measure-valued branching processes, Conference in Stochastic Analysis and Related Topics, Department of Mathematics, Purdue University, USA May 2015.

47. L. Beznea, invited talk: Measure-valued branching processes of Dirichlet and Neumann nonlinear boundary value problems, Probability Seminar, Dept. of Math., Univ. of California at San Diego, La Jolla, USA, May 2015.

48. L. Beznea, invited talk: An equivalence between the Dirichlet and the Neumann problem for the Laplace operator, Bielefeld Stochastic Summer School, Bielefeld, Germany, August 2015.

49. I. R. Ionescu, Continuum viscoplastic simulation of a granular column collapse: rheology and lateral wall effects, "Viscoplastic Fluids: From Theory to Applications", Banff (Canada), October 2015

50. I. R. Ionescu, Viscoplastic modeling of granular column collapse with pressure dependent rheology, *8th GRACM International Congress on Computational Mechanics*, Volos (Greece), July 2015

51. I. R. Ionescu, Shallow viscoplastic modeling of dense avalanches: from the onset to the dynamic flow, *Avalanches and Rupture Phenomena*, International workshop, Nancy, February 2015

52. I. R. Ionescu, Wave propagation and anisotropic damage, *séminaire Institut Physique du Globe, Paris*, January 2015.

53. I.R. Ionescu, Multi-scale anisotropic damage modeling and dynamic wave propagation, *International Plasticity Symposium*, Montego Bay, Jamaica, January 4-9, 2015 (key-note speaker)

54. G. Marionschi, Feedback stabilization of system for phase separation, XIII-ème Colloque Franco-Roumain de Mathématiques Pures et Appliquées, Iaşi, August 2016

55. L. Beznea, invited talk: An equivalence between the Dirichlet and the Neumann problem for the Laplace operator, *Academia Romana 150-Sesiune de Comunicări, Institutul de Matematica "Octav Mayer"*, Iaşi, March 2016

56. L. Beznea, invited talk: Procese de ramificare: ecuaţii neliniatre asociate şi aplicaţii, *Colocvii matematice, Sesiune dedicată aniversări a 150 de ani de la înfiinţarea Academiei Romane*, Bucureşti, April 2016

57. L. Beznea, invited talk: Measure-valued branching processes associated with Neumann nonlinear semiflows, *Current Issues in PDEs and Stochastics*, Bucharest, June 2016

58. L. Beznea, invited talk: Branching processes associated with Neumann nonlinear semiflows, in the Special Session: "Maths et planete Terre", XIII-ème Colloque Franco-Roumain de Mathématiques Pures et Appliquées, Iaşi, August 2016

59. O. Lupascu, invited talk: Nonlinear PDEs related to Markov processes on finite configurations, *Current Issues in PDEs and Stochastics*, Bucharest, June 2016

60. O. Lupascu, invited talk: Measure-valued processes, a stochastic model for avalanches, *The 6th Workshop Series on Mathematics*, Piteşti, June 2016

61. O. Lupascu, invited talk: Measure-valued Markov branching process and applications, *Stochastic Analysis and Applications Workshop*, Braşov, July 2016

62. I. R. Ionescu, invited talk of the Special Session "Modeles mathematiques et methodes numeriques en mecanique des milieux continus", XIII-ème Colloque Franco-Roumain de Mathématiques Pures et Appliquées, Iaşi, August 2016

2.3 Research stages, courses, scientific seminars, and summer schools

1. O. Lupaşcu made *two research stages at LSPM, Univ. Paris 13*, France, in the frame of her PhD program, 2012

2. O. Lupaşcu participed at *The Seminar Homogeneisation et Echelles Multiples, Laboratoire Jacques-Louis Lions*, Paris 6, France, 2012
http://www.ljll.math.upmc.fr/fr/seminaires/gt_homogeneisation.html

3. O. Lupaşcu participed at *Spring School in Probabiliy*, Dubrovnik, Croatia, April 2012

4. O. Lupaşcu participed at *Summer School in Mathematics*, Brussels, August 2013

5. O. Lupaşcu participed at *Colloque du Laboratoire d'Excellence SEAM*, Paris, June 2013

6. O. Lupaşcu made a scientific visit (collaboration) at *Institut Elie Carten*, Nancy, France, Mai 2013

7. O. Lupaşcu made *two research stages at LSPM, Univ. Paris 13*, France, in the frame of her PhD program, 2013

8. L. Beznea, Research stages on the project topics: at University of Bielefeld, Germany, scientific collaboration with the stochastic analysis team, coordinated by Prof. Dr. Michel Rockner, and at Imperial College Londra, scientific collaboration with Prof. Dr. Dan Crisan, 2014.

9. L. Beznea, Research visite at Institut de Mathématiques de Toulouse, France, and participation at "Conference in Honour of Dominique Bakry", 2015.

10. L. Beznea, Research stages on the project topics: at University of Bielefeld, Germany, scientific collaboration with the stochastic analysis team, coordinated by Prof. Dr. Michel Rockner, 2015.

11. L. Beznea, Research stages on the project topics: at Worcester Polytechnic Institute (WPI), Worcester, USA, scientific collaboration with Umberto Mosco and Bogdan Vernescu and at University of California at San Diego, USA, scientific collaboration with R.K. Getoor and P.J. Fitzsimmons, 2015.

12. L. Beznea, Research visite at Alfred Renyi Institute of Mathematics, Budapest, Hungary and participation at the conference "Stochastics and Interactions", 2015.

13. L. Beznea, Research visite at Universitatea "Al. I. Cuza" of Iasi, and participation at "The Eighth Congress of Romanian Mathematicians", 2015.

14. O. Lupaşcu, made *two research stages at LSPM, Univ. Paris 13* on the project topics, France, 2015.

15. O. Lupaşcu participed at *Regional Romanian French Summer School in Applied Mathematics*, 1-9 iulie 2015, Sinaia, România.

16. L. Beznea, Research visite at University Paris Nord, France, June 2016.

2.4 Organization of conferences, workshops, and scientific seminars

1. *Probability and Related Aspects*, May 22-26, 2012, Alba-Iulia, Romania http://www.imar.ro/ProbAlbaIulia2012/conf.php, L. Beznea and O. Lupaşcu were co-organizers

2. *International Conference on Complex Analysis and Related Topics. The 13th Romanian - Finnish Seminar*, June 26-30 2012, Ploiesti, Romania http://imar.ro/RoFinSem2012/conf.php, L. Beznea and O. Lupaşcu were co-organizers

3. *International Conference on internationala PDEs and Stochastic Processes. The 5th Workshop Series on Mathematics*, October 2012, Pitesti, http://imar.ro/PDE&StoPr-Pitesti12/conf.php, L. Beznea and G. Marinoschi were co-organizers

4. *Joint International Meeting of the Amer. Math. Soc. and Romanian Math. Soc.*, June 2013, Alba Iulia, L. Beznea, O, Lupaşcu and G. Marinoschi were co-organizers

5. *The Eighth Congress of Romanian Mathematicians*, Iaşi, June 2015, L. Beznea, I. R. Ionescu, O, Lupaşcu and G. Marinoschi were co-organizers http://www.imar.ro/congmatro8/conf.php

6. *Colloque Franco-Roumain en Théorie des Probabilités*, Bucharest, November 2015, L. Beznea and O. Lupaşcu were co-organizers http://imar.ro/organization/activities/standalone/coll_prob.html

7. *Workshop. Viscoplastic Fluids: From Theory to Application*, Banff International Research Station (Canada), October 2015, I. Ionescu was co-organizer http://www.birs.ca/events/2015/5-day-workshops/15w5071

8. *XIII-ème Colloque Franco-Roumain de Mathématiques Pures et Appliquées*, O. Lupaşcu was co-organizer of the Special Session: Processus stocastiques, Iaşi, August 2016

9. *XIII-ème Colloque Franco-Roumain de Mathématiques Pures et Appliquées*, L. Beznea was the president of the scientific committee, Iaşi, August 2016

3 Brief presentation of the results

• In the article [1] *Viscoplastic shallow flow equations with topography* the shallow flow of a viscoplastic fluid over a general basal topography is investigated. The plasticity (flow/no flow) criteria of the constitutive law may include Von - Mises (Bingham fluid) or

Drucker - Prager (Mohr - Coulomb) models. Coulomb frictional conditions on the bottom are considered. Assuming that the shear stresses are small with respect to the stresses associated to the tangent plane, an asymptotic analysis is developed for small thickness aspect ratio. A Saint-Venant model and a new depth integrated theory is presented. The resulting shallow flow equations have the same structure as the three dimensional ones. The 2-D (tangent) momentum balance law and the thickness evolution equation are closed with a "shallow constitutive equation" which links the averaged stresses to the rate of deformations in the tangent plane. The shallow flow / no flow (yield) condition and the shallow viscosity are not the same as in the three dimensional case but the constitutive law has the same structure. The curvature of the bottom surface is included in the model in the expression of the differential operators as well as in the frictional terms. To illustrate the capabilities of the shallow model to reproduce the flow, the sheet flow is analyzed. Two comparisons between the (2-D) in-plane channel flow and the asymptotic (1-D) flow for the Drucker-Prager fluids are considered. One comparison involves the experimental data and the other one includes ALE (Arbitrary Lagrangian-Eulerian) computations. A couple of boundary value problems, modeling shallow dense avalanches, for different viscoplastic laws are selected to illustrate the predictive capabilities of the model: spreading a Drucker-Prager dome on a talweg and the role of barriers in stopping a viscoplastic avalanche.

• In the paper [2] *Augmented Lagrangian for shallow viscoplastic flow with topography* we have developed a robust numerical algorithm for the visco-plastic Saint- Venant model with topography. For the time discretization an implicit (backward) Euler scheme was used. To solve the resulting nonlinear equations, a four steps iterative algorithm was proposed. To handle the non-differentiability of the plastic terms an iterative decomposition-coordination formulation coupled with the augmented Lagrangian method was adopted. The proposed algorithm is consistent, i.e. if the convergence is achieved then the iterative solution satisfies the nonlinear system at each time iteration. The equations for the velocity field are discretized using the finite element method, while a discontinuous Galerkin method, with an upwind choice of the flux, is adopted for solving the hyperbolic equations that describe the evolution of the thickness. The algorithm permits to solve alternatively, at each iteration, the equations for the velocity field and for the thickness. The iterative decomposition coordination formulation coupled with the augmented Lagrangian method works very well and no instabilities are present. The proposed algorithm has a very good convergence rate, with the exception of large Reynolds numbers (Re¡¡ 1000), not involved in the applications concerned by the shallow viscoplastic model.The discontinuous Galerkin technique assure the mass conservation of the shallow system. The model has the exact C-property for a plane bottom and an asymptotic C-property for a general topography. Some boundary value problems were selected to analyze the robustness of the numerical algorithm and the predictive capabilities of the mechanical model. The comparison with an exact rigid flow solution illustrates the accuracy of the numerical scheme in handling the non-differentiability of the plastic terms. The influence of the mesh and of the time step are investigated for the flow of a Bingham fluid in a talweg. The role of the material cohesion in stopping a viscoplastic avalanche on a talweg with barrier was analyzed. Finally, the capacities of the model to describe the flow of a Bingham fluid on

a valley from the broken wallof a reservoir situated upstream were investigated.

• In the paper [3] *Optimization of a singular flow in a porous medium*, we address an optimization problem related to a diffusive flow in a nonhomogeneous porous medium. More exactly, the purpose is to control by a flow parameter an optimal average of the solution in a subset of the domain. The state model is degenerate and ill-posed and this requires to develop a technique of optimization for a singular state system.

• In the paper [4] *A unifying construction for measure-valued continuous and discrete branching processes*, we first emphasize conditions for a sub-Markovian resolvent of kernels to ensure that it is associated to a càdlàg Markov process. It turns out that this is a method of proving existence and path regularity for both continuous and discrete measure-valued branching Markov processes.

• In lucrarea [5] *Perturbation methods and identification problems for degenerate evolution systems* se determina conditiile in care se poate rezolva problema identificarii unui termen sursa intr-o ecuatie parabolica degenerata cu operatori liniari inchisi intr-un spatiu Banach, utilizand o informatie suplimentara asupra solutiei. Lucrarea extinde rezultatele anterioare ale autorilor cu privire la tratarea acestui tip de probleme pentru ecuatii hiperbolice si sisteme de ecuatii degenerate.

• In the paper [6] *Subordination in the sense of Bochner of L^p-semigroups and associated Markov processes* we show that the subordination induced by a convolution semigroup (subordination in the sense of Bochner) of a C_0-semigroup of sub-Markovian operators on an L^p space is actually associated to the subordination of a right (Markov) process. As a consequence, we solve the martingale problem associate with the L^p-infinitesimal generator of the subordinate semigroup. We also prove quasi continuity properties for the elements of the domain of the L^p generator of the subordinate semigroup. It turns out that an enlargement of the base space is necessary. A main step in the proof is the preservation under such a subordination of the property of a Markov process to be a Borel right process. We use several analytic and probabilistic potential theoretical tools.

• In the article [7] *A control problem for a cross-diffusion system in a nonhomogeneous medium*, we study a model, motivated by a bioremediation process, describing a cross-diffusion movement of a bacteria population b attracted by a chemoattractant signal c, in a nonhomogeneous stratified medium with n layers. We assume that this reaction-diffusion process is characterized by a low rate of degradation and a low diffusion coefficient of the chemoattractant, expressed in the model by a small parameter ϵ. The model consists of n systems of nonlinear parabolic equations with transmission conditions between layers. We prove a global-in-time solution for the asymptotic model setup with respect to the small parameter of the problem, for arbitrarily large initial data. Next, we deal with the control problem focusing mainly on the reduction of the chemoattractant concentration, by acting upon the initial distribution of the bacteria population b_0. To this end, we prove the existence of a solution to the control problem and determine the optimality conditions.

• In the paper [8] *Bounded and L^p-weak solutions for nonlinear equations of measure-valued branching processes*, was studied the nonlinear Schrödinger type equation $(\Delta - c)u + c\sum_{k=1}^{\infty} q_k u^k = f$, where c is a bounded positive function. We may replace the Laplace operator with the generator of a Borel right Markov process with state space E and consider a nonlinear term induced by a perturbing kernel. We solve this type of equation in an L^p-weak sense, involving positive definite extensions of the solution from E to the set of all finite positive measures on$E$. The extensions to the set of measures are related to the classical connection (from the works of M. Nagasawa, N. Ikeda, S. Watanabe, and M.L. Silverstein) between this nonlinear equation and the discrete branching Markov processes. We also investigate the existence of the bounded solutions for the Dirichlet problem associated to the above equation, with bounded boundary data. Our approach is based on probabilistic and analytic potential theoretical methods, used on both spaces E and the set of all finite configurations of E, like the Revuz formula for the continuous additive functionals and the perturbation with kernels of the transition functions and resolvents.

• In the paper [9] *On the existence of the dual right Markov process and applications* was proved that given a Borel right process there exists a dual process which is also a right Markov process. However, it is necessary to enlarge the initial space to a new Lusin topological space and the dual process is a right process with respect to a second Lusin topology. As a result both processes can be identified as solutions to martingale problems. Another application is the proof that the Riesz decomposition holds (in potential and harmonic components) for the excessive functions and the set of all potentials becomes solid in order, that completes a recent result of P.J. Fitzsimmons and R.K. Getoor.

• In the paper [10] *Control approach to an ill-posed variational inequality* it is studied the bounday problem which describes an absorption-desorption process (e.g., a fuid absorbed into a porous material, the oxygen absorbed by biological tissues, or the radiation retained or ceded by materials or tissues). It is proved here the existence of a generalized solution of the problem, using an optimal control technique and numerical simulations are established.

• In the article [11] *A control approach for an identification problem associated to a strongly degenerate parabolic system with interior degeneracy* identifucation problems are studied, based on some observations, on the diffusion coefficient for a degenerate parabolic equation, with internal degeneracy. Two identification problems are investigated, and for both cases it is proved the existence of the solution of the states system, the existence of a solution for the minimizing problem, and the optimality conditions are computed.

• In the paper [12] *Modeling shallow avalanche onset over complex basal topography* was modeled of the onset of a shallow avalanche (soils, snow or other geomaterials) over various bottom topologies (mountains, valleys, ...). In order to do that, we use the shallow visco-plastic model with topography, developed in [I.R. Ionescu, Viscoplastic shallow flow equations with topography, Journal of Non-Newtonian Fluid Mechanics, 193, 116–128 (2013)], and we introduce a simple criterion to distinguish if an avalanche occurs or

not. This criterion, relating the yield limit (material resistance) to the distribution of the external forces, is deduced from an optimization problem, called limit load analysis. The plastic dissipation functional which is involved in the limit load problem is non smooth and non coercive in the classical Sobolev spaces. To prove the existence of an onset velocity field (collapse flow) the appropriate functional space consists of bounded tangential deformation functions. We propose therefore a numerical strategy to solve the limit load problem and to get the onset flow field. A mesh free method, called the discontinuous velocity domain splitting (DVDS), is adapted here. The limit load problem is thus reduced to the minimization of a shape dependent functional. The discontinuous collapse flow velocity field is associated to a sub-domain and a rigid flow. With a level set of a Fourier function we give a description of the sub-domains and then we use genetic algorithms to solve the resulted non convex and non smooth global optimization problem. Finally, we illustrate the proposed numerical approach by solving several safety factor problems.

• In the paper [13] *Measure-valued discrete branching Markov processes* was studied and constructed branching Markov processes on the space of finite configurations of the state space of a given standard process, controlled by a branching kernel and a killing one. In particular, we may start with a superprocess, obtaining a branching process with state space the finite configurations of positive finite measures on a topological space. A main tool in proving the path regularity of the branching process is the existence of convenient superharmonic functions having compact level sets, allowing the use of appropriate potential theoretical methods.

• In the paper [14] *Onset of a dense avalanche on a plane slope* is studied the safety factor (of limit load) problem related to the shallow flow avalanche of a visco-plastic fluid/solid with heterogeneous thickness over a plane slope. The first objective is to find the appropriate functional space of the problem and to prove the existence of a onset velocity field. The second objective of the paper is to propose a numerical strategy to solve the limit load problem and to characterize the flow onset of the avalanche. We introduce an optimization problem (called the limit load or safety factor problem) to study the link between the yield limit, the external forces and the thickness distributions for which the shallow flow avalanche of a visco-plastic fluid/solid does, or does not occur. This optimization problem is reconsidered in the space of bounded deformations functions and the velocity boundary conditions are relaxed. We prove that the initial optimization problem is not changed and the reformulated safety factor problem has a least a solution, modeling the onset of the avalanche. We have developed here a DVDS-type numerical technique to solve the safety factor problem through a shape optimization problem. The proposed numerical method makes use of Fourier level set description of the subdomain and of a genetic algorithm in solving the non convex and non-smooth global optimization problem. The proposed numerical approach is illustrated with some numerical simulations of avalanches involving a Bingham circular dome, a Druker-Prager square dome on a plane slope, and a thick Bingham fluid over an obstacle.

• The paper [15] *A boundary control problem for a possibly singular phase field system with dynamic boundary conditions* deals with an optimal control problem related to

a phase field system of Caginalp type with a dynamic boundary condition for the temperature. The control placed in the dynamic boundary condition acts on a part of the boundary. The analysis carried out in this paper proves the existence of an optimal control for a general class of potentials, possibly singular. The study includes potentials for which the derivatives may not exist, these being replaced by well-defined subdifferentials. Under some stronger assumptions on the structure parameters and on the potentials (namely for the regular and the logarithmic case having single-valued derivatives), the first order necessary optimality conditions are derived and expressed in terms of the boundary trace of the first adjoint variable.

• In the paper [16] *Onset of a dense avalanche on a plane slope* the main goal is to study the safety factor (of limit load) problem related to the shallow flow avalanche of a visco-plastic fluid/solid with heterogeneous thickness over a plane slope. The first objective is to find the appropriate functional space of the problem and to prove the existence of a onset velocity field. The second objective of the paper is to propose a numerical strategy to solve the limit load problem and to characterize the flow onset of the avalanche. We introduce an optimization problem (called the limit load or safety factor problem) to study the link between the yield limit, the external forces and the thickness distributions for which the shallow flow avalanche of a visco-plastic fluid/solid does, or does not occur. This optimization problem is reconsidered in the space of bounded deformations functions and the velocity boundary conditions are relaxed. We prove that the initial optimization problem is not changed and the reformulated safety factor problem has a least a solution, modelling the onset of the avalanche. We have developed here a DVDS-type numerical technique to solve the safety factor problem through a shape optimization problem. The proposed numerical method makes use of Fourier level set description of the subdomain and of a genetic algorithm in solving the non convex and non-smooth global optimization problem. The proposed numerical approach is illustrated with some numerical simulations of avalanches involving a Bingham circular dome, a Druker-Prager square dome on a plane slope, and a thick Bingham fluid over an obstacle.

• The paper [17] *Viscoplastic modelling of granular column collapse with pressure dependent rheology* propose to reproduce laboratory experiments of granular column collapse over inclined planes, depending on both the pressure p and the norm of the strain rate tensor kDk. 2-D simulations using this model well reproduce the dynamics and deposits of collapsing granular columns. The computed runout distances and slopes of thedeposits agree very well with the values found in the experiments. The observed slumping behavior therefore appears to be mainly due to the flow/no-flow criterion and to the associated strain-independent part of the flowing constitutive relation (i.e. related to plastic effects).

• In the paper [18] *An equivalence between the Dirichlet and the Neumann problem for the Laplace operator* we give a representation of the solution of the Neumann problem for the Laplace operator on the n-dimensional unit ball in terms of the solution of an associated Dirichlet problem. The representation is extended to other operators besides the Laplacian, to smooth simply connected planar domains, and to the infinite-dimensional Laplacian on the unit ball of an abstract Wiener space, providing in particular an explicit

solution for the Neumann problem in this case. As an application, we derive an explicit formula for the Dirichlet-to-Neumann operator, which may be of independent interest.

• In the paper [19] *Stochastic equation of fragmentation and branching processes related to avalanches* we introduce a stochastic model for the fragmentation phase of an avalanche. We construct a fragmentation-branching process related to the avalanches, on the set of all fragmentation sizes introduced by J. Bertoin. A fractal property of this process is emphasized. We also establish a specific stochastic differential equation of fragmentation. The results are obtained by combining analytic and probabilistic potential theoretical tools.

• The article [20] *Fish populations dynamics with nonlinear stock-recruitment renewal conditions* The dynamics of a fish population with age-structure and space diffusion is studied under a renewal condition represented by various nonlocal nonlinear stock-recruitment functions, instead of the standard linear birth condition. This population dynamics model is approached as a Cauchy problem for an evolution equation with an unbounded nonlinear operator in a Hilbert space. The domain of the operator contains specific restrictions induced by the definition of the stock-recruitment function which make not possible the proof of the m-accretiveness property. Its lack is compensated by some other essential properties proved in the paper, which allow the proof of the existence and uniqueness of the solution. The semigroup formulation of the problem ensures the convergence of a time-difference scheme used for providing some numerical simulations which can give information about the stock, recruitment and fishing strategy.

• In the paper [21] *A singular nonconvex optimal control problem*, we address a nonconvex optimal control problem governed by a nonlinear elliptic equation with a discontinuous diffusion term. This optimal control problem may be viewed as singular in the sense of Lions We refer to phenomena related to the equilibrium of continuous media, material science, strength of materials.

• In the article [22] *Measure-valued branching processes associated with Neumann nonlinear semiflows* we construct a measure-valued branching Markov process associated with a nonlinear boundary value problem, where the boundary condition has a nonlinear pseudo monotone branching mechanism term. The process is then used in the probabilistic representation of the solution of the parabolic problem associated with a nonlinear Neumann boundary value problem. In this way the classical association of the superprocesses to the Dirichlet boundary value problems also holds for the nonlinear Neumann boundary value problems. It turns out that the obtained branching process behaves on the measures carried by the given open set like the linear continuous semiflow, induced by the reflected Brownian motion, while the branching occurs on the measures having non-zero traces on the boundary of the open set.

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