

**Ngaiming Mok: From rigidity phenomena on proper holomorphic maps
to the geometric theory of minimal rational curves**

Abstract: This lecture concerns Function Theory on bounded symmetric domains and their generalizations on the one side, and rational curves on projective manifolds on the other. We establish a link between the two through the study of rigidity problems on proper holomorphic maps.

The seminal work of Shigefumi Mori yielded rational curves on a projective manifold whose canonical bundle is not numerically effective, leading by Miyaoka-Mori to the proof that Fano manifolds are uniruled, i.e., covered by rational curves. In the last ten years or so, the speaker and Jun-Muk Hwang have developed a geometric theory on uniruled projective manifolds basing on the study of vectors tangent to free rational curves of minimal degree, called *varieties of minimal rational tangents* (VMRTs). One of the key principles established in the geometric theory is the Cartan-Fubini extension principle, according to which a germ of local biholomorphism two Fano manifolds of Picard number 1 extends automatically to a global biholomorphism unless the VMRT at a general point is a finite union of projective linear subspaces.

Recently, in examining the problem of rigidity of proper holomorphic maps between irreducible bounded symmetric domains of rank ≥ 2 , a problem posed by the speaker and resolved by Tsai using methods of Kähler Geometry and Lie Theory, the speaker has found an intimate link between the study of proper holomorphic maps and the geometric theory of uniruled projective manifolds based on VMRTs. This link is realized by identifying a bounded symmetric domain as a domain in its compact dual, e.g., a Type-I domain as a domain on the Grassmann manifold. Harmonic Analysis on bounded symmetric domains leads to differential constraints which translate the rigidity problem to a question of characterizing germs of *non-equidimensional* holomorphic immersions which send VMRTs into linear sections of VMRTs. The speaker has settled the latter problem in terms of a *relative* differential projective-geometric criterion concerning the second fundamental form on VMRTs. Most recently, in a joint work with Jaehyun Hong we have extended the non-equidimensional Cartan-Fubini extension principle for germs of holomorphic immersions between general uniruled projective

manifolds under the same condition on the second fundamental form, and obtained as a consequence the characterization of certain standard embeddings between rational homogeneous manifolds of Picard number 1 in terms of VMRTs.