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Transcendental methods in Complex Analysis

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ABSTRACT

The subject of this thesis is analytic pseudoconvexity and it is devoted to problems concerning Stein spaces, q-convexity and integral representations.

The first part of the thesis, represented by the first two chapters, includes general notions about complex spaces, normal spaces, Stein spaces, q-convexity and q-convexity with corners. It also contains some definitions and results needed for the second part of the thesis: Colţoiu-Diederich theorem which relates the Levi problem to the existence of the envelope of holomorphy, Skoda's theorem, Colţoiu's theorem concerning the finite dimension of certain cohomology groups and, in the end, Grauert and Lieb's theorem on the construction of a Ramírez kernel which yields the existence of bounded solutions to the $\overline{\partial}$-equation on bounded, strictly pseudoconvex domains.

The second part, consisting of the last three chapters (3,4, and 5), contains the original results of the thesis. These theorems are included in author's papers [19], [20], and [21].

The main theorem from chapter 3 generalizes a theorem of Fornæss and Narasimhan, related to the Levi problem in normal spaces, by showing that it remains valid even without the assumption of relative compactness. It is then applied to prove a result concerning the Serre problem, which says that a necessary and sufficient condition for a locally trivial fiber bundle X, with Stein base and bounded domain of holomorphy in $\mathbb{C}^n$ as fiber, to be Stein is the existence of the envelope of holomorphy for X. A proof of this theorem, in a particular case only and with a much more difficult proof was given by Zaffran, using Inoue-Hirzebruch surfaces.

The central theorem from chapter 4 is a result about the cohomological properties of the intersection of $(n-1)$-complete open subsets in $\mathbb{C}^n$: the transversal intersection of finitely many $(n-1)$-complete, bounded open subsets with $\mathcal{C}^2$ boundary in $\mathbb{C}^n$ is cohomologically $(n-1)$-complete.

In the last chapter it is proved a result which gives a positive answer to the Corona problem on strictly pseudoconvex, bounded domains in $\mathbb{C}^n$, when an extra condition is verified by the corona data $f_1, \dots, f_k$: it is assumed that

there exist $i \neq j$ such that the sublevel sets $\{|f_i| < \alpha\}$ and $\{|f_j| < \alpha\}$ are separated near the boundary.

Further on, these main new results of the thesis will be presented with more details.

The following theorem is the main result of chapter 3:

Theorem 1. *Let X be a normal Stein space and $\Omega \subset X$ an open, locally Stein subset of X. Then, for every sequence of points $(x_n)_n$ in Ω which tends to a limit $x \in \partial\Omega \setminus X_{\text{sing}}$, there exists a holomorphic function on Ω which is unbounded on $(x_n)_n$.*

This theorem generalizes a result by Fornæss and Narasimhan [10], by dropping the assumption of relative compactness of Ω. The proof follows the same steps as the one given by Fornæss and Narasimhan, but each step is proven using different arguments, which carefully avoid using the relative compactness of Ω.

The strategy for the proof is the following: we choose a point $p \in \partial\Omega \setminus X_{\text{sing}}$ and $h_1, \ldots, h_m$ holomorphic functions on X with p the only common zero. We define the function $f : X \to \mathbb{C}$ by $f(x) = \det((v_i \Phi_j(x))_{i,j=1,\ldots,n})$, where $(\Phi_1, \ldots, \Phi_n)$ are the components of a holomorphic map $\Phi : X \longrightarrow \mathbb{C}^n$ with branch locus Z', such that Φ has discrete fibers and $p \notin Z'$, and $v_1, \ldots, v_n$ are holomorphic vector fields on X, generating the tangent space of X at p. $Z = \{f = 0\}$ is a hypersurface of X which does not contain p.

Then, a modified version of a theorem by Henri Skoda [25, Thm.1] provides us with m holomorphic functions $g_1, \ldots, g_m$ on $\Omega \setminus Z$ with the sum of their modules going to infinity near p. Using the normality of the space and the good properties of the holomorphic function f, the functions fg_i will be extended to holomorphic functions on Ω. Then, given a sequence of points in Ω converging to p, at least one of these functions is unbounded on this sequence, which ends the proof.

This generalized theorem can be applied to obtain a characterization theorem for a particular case of the Serre problem, which (in the general case) asks whether a locally trivial holomorphic fiber bundle with Stein base and Stein fiber, is Stein. Since Serre proposed this problem in 1953, various counterexamples and special cases have been proved.

For the particular case of bounded Stein domain in $\mathbb{C}^n$ as fiber, and with an additional assumption on the fiber, Siu [24] proved that the problem has positive answer. The first counterexample to the Serre problem was given in 1977 by Skoda [26], who constructed a holomorphic fiber bundle which is not Stein, with $\mathbb{C}^2$ as fiber and a domain of $\mathbb{C}$ as base. Later, Demailly [8] improved this counterexample by showing that one can choose the base of the fiber to be the complex plane $\mathbb{C}$ or the unit disc $\mathbb{D} \subset \mathbb{C}$. Another counterexample was given in 1985 by G.Coeuré and J.J.Loeb [4]: they solved negatively the Serre problem for the particular case of a locally trivial bundle with Stein base and bounded Stein domain in $\mathbb{C}^n$ as fiber. Studying the example of Coeuré and Loeb, Pflug and Zwonek [18] gave a characterization of hyperbolic Reinhardt domains in $\mathbb{C}^2$ which can be used as fibers for a counterexample to the Serre problem. In 2001, D.Zaffran [27] proved, using Hirzebruch-Inoue surfaces, that the counterexample of Coeuré and Loeb does not admit envelope of holomorphy. Related to this, as an application of Theorem 1, we prove the following characterization theorem:

Theorem 2. *Let $p : S \longrightarrow B$ a localy trivial bundle with Stein base B and bounded domain of holomorphy $F \subset \mathbb{C}^n$ as fiber. Then, S is Stein iff S has envelope of holomorphy.*

This theorem has a very nice and short proof, which uses Theorem 1: Denote by Z the envelope of holomorphy of S. By a result of Siu [24], global functions on S separate points and give local coordinates. Hence, S is open in Z. Since B is Stein, it can be embedded as a closed submanifold of $\mathbb{C}^k$ defined by global holomorphic functions. Consequently, p can be holomorphically extended to $p_1 : Z \longrightarrow B$. For every sufficiently small Stein open subset U of B such that on U the bundle is trivial, $p_1^{-1}(U)$ is an open subset and $p_1^{-1}(U) \cap S = U \times F$, where F is a bounded Stein domain in $\mathbb{C}^n$. Moreover, every boundary point $x \in \partial S$ is contained in one of the sets $p_1^{-1}(U)$ above. Thus, S is locally Stein in Z. Now, using a generalized version of the theorem of Colţoiu and Diederich [7] (by dropping the assumption of relative compactness from the hypothesis, which is possible, thanks to Theorem 1), we may conclude that S is Stein. The reversed implication is trivial. In this way, a more general result than Zaffran's theorem was obtained, with a much simpler proof.

In chapter 4, the coomologic properties of finite intersections of $(n-1)$-complete open subsets of $\mathbb{C}^n$ are investigated.

Diederich and Fornæss [9] showed that any q-convex function with corners on a complex manifold can be uniformly approximated with $\tilde{q}$-convex functions, where $\tilde{q} = n - \lfloor \frac{n}{q} \rfloor + 1$. Combining this with the results in [1], one obtains, in particular, that every finite intersection of q-complete open subsets of a complex manifold is cohomologically $\tilde{q}$-complete. A partially improved result was obtained by Matsumoto [17], who introduced the integer $\hat{q} = n - \lfloor \frac{n-1}{q} \rfloor$, which verifies $\hat{q} = \tilde{q}$ if $q|n$ and $\hat{q} = \tilde{q} - 1$ if $q \not| n$ and proved that for every finite intersection $\cap_{j=1}^{t} D_j$ of q-complete open subsets of a non-compact complex manifold M, we have $H^i(\cap_{j=1}^{t} D_j, \mathcal{F}) = 0$, for every $\mathcal{F} \in \mathrm{Coh}(M)$ and every $i \geq \hat{q}$.

However, for $q = n - 1$ theorems from [9] are of no use, since $\tilde{q} = n$ and we already know from the Greene-Wu [14] theorem that any noncompact complex n-dimensional manifold is n-complete.

Also, for $q = n - 1$, Matsumoto's integer $\hat{q} = n - 1$, but hers theorem does not prove cohomologic $(n-1)$-completeness, since it requires the seaf $\mathcal{F}$ to be coherent on the whole space M and not only on the intersection $\cap D_j$.

In the survey [6], Colţoiu states the following problem: If D is a $(n-1)$-complete with corners, open subset of $\mathbb{C}^n$, does it follow that D is $(n-1)$-complete? A weaker version of this problem is to study whether or not every $(n-1)$-complete with corners, open subset of $\mathbb{C}^n$ is cohomologically $(n-1)$-complete. A particular case of $(n-1)$-complete with corners, open subsets of $\mathbb{C}^n$ is represented by finite intersections of $(n-1)$-complete open subsets of $\mathbb{C}^n$. In the chapter 4 of the thesis, it is shown that, under additional assumptions for the boundaries of the open sets which form the intersection, this particular problem has affirmative answer.

The proof uses the method developed by Colţoiu for proving the W.Barth conjecture [5]. Firstly, the follwing lemma concerning topological properties is proved:

Lemma 3. *Consider $D_j \subset \mathbb{C}^n$, $j = 1, \dots, r$, bounded, open and with $\mathcal{C}^2$-boundary such that every two boundaries intersect transversally.*

Then, there exists constants $d_0 > 0$ and $\eta_0 > 0$ sufficiently small with the following property: for any $\tau_1, \dots, \tau_r \in \mathcal{C}_0^\infty(\cap_{j=1}^{r} D_j)$, $\tau_1 \geq 0, \dots, \tau_r \geq 0$, there is a sufficiently small constant $\lambda_0 = \lambda_0(\tau_1, \dots, \tau_r) > 0$ such that for any constants $0 \leq \mu_j \leq \lambda_0$, $j = 1, \dots, r$, the set

$$B_{ij}(d) = (D_i \cup D_j) \setminus \\ \setminus \left(\{\delta_{\partial D_i}(x) e^{-\eta_0 \|x\|^2 - \mu_i \tau_i(x)} > d\} \cup \{\delta_{\partial D_j}(x) e^{-\eta_0 \|x\|^2 - \mu_j \tau_j(x)} > d\} \right)$$

has no compact, connected components for any $1 \leq i, j \leq r$ and for any $0 < d < d_0$.

Although its statement is similar to [5, Lemma 2.4], the proof is different. It uses the strong deformation retractions given by the Collar Neighborhood

Theorem for manifolds with boundary, for three different subsets of $\overline{B_{ij}(d)}$, which are then glued together to obtain a global strong deformation retraction of $\overline{B_{ij}(d)}$ into $\partial(D_1 \cup D_2)$, which implies that $B_{ij}(d)$ has no compact, connected components.

The next lemma is an adaptation of [5, Lemma 3.5] to the context and notations of the problem stated above. It uses the approximation result [1, p.250], with the difficult part being to prove the approximation of the $(n-2)$-cohomology groups, needed for applying Lemma [1, p.250]. This approximation holds only for d sufficiently small, given by Lemma 3.

Lemma 4. *Let $D_j \subset\subset \mathbb{C}^n$, $1 \leq j \leq r$, be $(n-1)$-complete open subsets with $\mathcal{C}^2$ boundary, such that every two boundaries intersect transversally.*

Then, there exist a constant $c_0 > 0$ and $(n-1)$-convex, exhaustion functions $\varphi_j : D_j \to \mathbb{R}$, such that, denoting $\varphi = \max(\varphi_1, \ldots, \varphi_r)$, for every $\mathcal{F} \in \mathrm{Coh}(\cap_{j=1}^r D_j)$ and any $c > c_0$, the restriction map

$$H^{n-1}(\cap_{j=1}^r D_j, \mathcal{F}) \to H^{n-1}(\{\varphi < c\}, \mathcal{F})$$

is bijective. In particular, $\cap_{j=1}^r D_j$ is cohomologically $(n-1)$-convex.

It is a simple exercise to prove that in a Stein space, and in particular in $\mathbb{C}^n$, q-convexity and q-completeness are equivalent. The next lemma, inspired from Ballico's article [2], shows, using induction on the dimension of the space and an argument by contradiction, that in holomorphically separate complex spaces, and in particular in $\mathbb{C}^n$, cohomological q-convexity and cohomological q-completeness are equivalent.

Lemma 5. *In a holomorphically separate complex space X with $\dim(X) < \infty$, every cohomologically q-convex open subset $U \subset X$ is cohomologically q-complete.*

As a direct consequence of the last two lemmas, the main theorem of chapter 4 is obtained:

Theorem 6. *Let $D_j \subset\subset \mathbb{C}^n$, $1 \leq j \leq r$, be $(n-1)$-complete open subsets with $\mathcal{C}^2$ boundary such that every two boundaries intersect transversally. Then, $\cap_{j=1}^r D_j$ is cohomologically $(n-1)$-complete.*

The chapter ends with a proof which shows that the example of domain in $\mathbb{C}^n$ given by Diederich and Fornæss in [9] for proving the optimality of $\tilde{q}$

verifies the conditions in our theorem when $q = n-1$, therefore it is a cohomologically $(n-1)$-complete domain on which there exists an $(n-1)$-convex with corners, exhaustion function which cannot be approximated by $(n-1)$-convex functions. In view of the reversed implication for the Andreotti-Grauert theorem [1], which is an open problem, it is worth studying if this example given by Diederich and Fornæss is $(n-1)$-complete.

The last chapter of the thesis, chapter 5, is devoted to studying a particular case of the Corona problem in strongly pseudoconvex domains in $\mathbb{C}^n$.

The commutative Banach algebra and Hardy space $H^\infty(\mathbb{D})$ consists of the bounded holomorphic functions on the open unit disk $\mathbb{D}$. Its spectrum S contains $\mathbb{D}$, because for any $z \in \mathbb{D}$, there is a maximal ideal which consists of all functions $f \in H^\infty(\mathbb{D})$ with $f(z) = 0$. The subspace $\mathbb{D}$ cannot make up the entire spectrum S, essentially because the spectrum is a compact space and $\mathbb{D}$ is not. It was conjectured by S. Kakutani in 1941 that the complement of the closure of $\mathbb{D}$, called the corona, is empty (this is known as the *corona problem*). It turns out that this is equivalent to a more elementary statement: *Suppose that $f_1, \ldots, f_k$ are bounded, holomorphic functions on the unit disk $\mathbb{D}$, with the property that*

$$|f_1(\zeta)| + |f_2(\zeta)| + \ldots + |f_k(\zeta)| > \delta > 0$$

for some positive constant δ and all $\zeta \in \mathbb{D}$ (we call $f_1, \ldots, f_k$ a set of corona data on $\mathbb{D}$). Then, there exist bounded, holomorphic g_j, $j = 1, \ldots, k$, on $\mathbb{D}$ such that

$$f_1(\zeta)g_1(\zeta) + f_2(\zeta)g_2(\zeta) + \ldots + f_k(\zeta)g_k(\zeta) \equiv 1$$

for all $\zeta \in \mathbb{D}$.

Lennart Carleson [3] solved affirmatively the corona problem on the unit disc D in $\mathbb{C}$ in 1962. Since then, the problem has been studied on arbitrary domains in $\mathbb{C}$ and also in several variables. It has been proved to have positive solution for various domains in $\mathbb{C}$ (see, for example, [12]). Fornæss and Sibony [11] and [23] have investigated the corona problem in several variables and constructed some pseudoconvex domains for which the problem does not hold true, one example being a bounded domain with strongly pseudoconvex boundary, except for one point. However, for domains with strongly pseudoconvex boundary, such an example has not been found and even for simple domains, such as the unit ball in $\mathbb{C}^n$, the problem is still open. Until now, there is no domain in the complex plane on which the problem is known to fail, and no pseudoconvex domain in $\mathbb{C}^n$ on which the problem is known to hold.

In this last chapter, the Corona problem on bounded, strongly pseudoconvex domains in $\mathbb{C}^n$ is studied, when an extra condition is satisfied by the corona data. The goal is to generalize in some way the results of Krantz [16] to an arbitrary number of pieces of corona data, using the same method and obtain, in this setting, a solution to the problem. Chapter 5 ends with some remarks about the necessity of a stronger extra condition than the one used in [16], even for two pieces of data.

The main result is the following:

Theorem 7. *Let $\Omega \subset\subset \mathbb{C}^n$ be a bounded, strongly pseudoconvex domain and $f_1, f_2, \ldots, f_k$ corona data on Ω. Let $V_j(\alpha) = \{z \in \Omega : |f_j(z)| < \alpha\}$. Assume that there exist $\alpha > 0$ and $i, j \in \{1, \ldots, k\}$ such that*

$$\overline{V_i(\alpha)} \cap \overline{V_j(\alpha)} \cap \partial\Omega = \emptyset,$$

where the closures are taken in $\mathbb{C}^n$.

Then, there exist bounded, holomorphic $g_1, g_2, \ldots, g_k$ on Ω such that

$$\sum_j f_j g_j \equiv 1.$$

The strategy for the proof is the following: If $f_1, f_2, \ldots, f_k$ are given corona data on Ω, we define $U_j(\alpha) = \{\zeta \in \Omega : |f_j(\zeta)| > \alpha\}$. Then, it is clear that $U_j(\alpha)$ are open and $\bigcup_j U_j(\alpha) \supseteq \Omega$ for any $\alpha < \delta/2k$, where δ is the constant from the corona condition. Let $(\varphi_j)_j$ be an arbitrary partition of unity subordinated to the covering $(U_j(\alpha))_j$. Now set

$$g_j = \frac{\varphi_j}{f_j} + \sum_i v_{ji} f_i,$$

where v_{ji} are functions to be determined later in the proof, with the properties $v_{ij} = -v_{ji}$ and $v_{ii} = 0$. Then, a simple formal verification shows that g_j are well-defined and $\sum_j f_j g_j \equiv 1$. Because φ_j are real functions, the functions g_j so defined are not necessarily holomorphic, but it can be choosen a convenient partition of unity $(\varphi_j)_j$ for which it is possible to choose v_{ji} such that each g_j will be holomorphic and bounded.

The holomorphicity of g_j is equivalent to $\overline{\partial} g_j = \frac{\overline{\partial}\varphi_j}{f_j} + \overline{\partial}\left(\sum_i v_{ji} f_i\right) \equiv 0$, and we denote by $S(k) = S(k; \varphi_1, \ldots, \varphi_k)$ this system of k equations. Inductively, it can be proved that it has bounded solutions v_{ij}, by extending at every step, at a larger domain, the bounded solutions from the previous step. For the induction step, the existence of bounded solutions for the $\overline{\partial}$-equation with bounded coefficients on strongly pseudoconvex domain with piecewise smooth boundary, proved by [15], is used. The only different step is the one next to the last, where a lemma based on the construction by Grauert and Lieb [13] of a Ramírez integral kernel [22] is used instead: if D is a bounded, strongly pseudoconvex open subset of $\mathbb{C}^n$ and $W(\alpha') = \{z \in D : \gamma(z) < \alpha'\}$, where γ is a strongly plurisubharmonic function, and f is a $\overline{\partial}$-closed $(0,1)$-form in $W(\alpha')$ with $\operatorname{supp}(f)$ relatively compact in D and with bounded coefficients, then the equation $\overline{\partial}u = f$ has bounded solutions on $W(\alpha) = \{z \in D : \gamma(z) < \alpha\}$, where $\alpha < \alpha'$.

The extra condition that is assumed in the theorem, for two pieces of data, says that the sublevel sets $V_1(\alpha)$ and $V_2(\alpha)$ must be separated for a small positive constant α.

A more general case of this extra condition would be to consider functions f_1, f_2 for the corona data for which their zeroes are separated. Unfortunately, this is not enough for the existence of a partition of unity $(\varphi_j)_j$ subordinated to the covering $(U_j(\alpha))_j$, which has the (even weaker) property $|\nabla\varphi_j(z)| \leq C d_{\partial\Omega}(z)^{-1/m}$, where $C > 0$ is a constant, $d_{\partial\Omega}(z)$ is the euclidean distance from z to $\partial\Omega$ and $m > 2n + 3$, as it is assumed in [16]. In fact, for $m > 2$, this is no longer possible, so the method used in [16] for proving the existence of bounded solutions to the $\overline{\partial}$-equation fails. A simple counterexample constructed using infinite Blaschke products on a disk in $\mathbb{C}$ proves this, in last section of Chapter 5.

BIBLIOGRAPHY

[1] Andreotti, A.; Grauert, H.: *Théorèmes de finitude pour la cohomologie des espaces complexes*, Bull. Soc. Math. France **90** (1962) 193–259.

[2] Ballico E.: *Annullamento di gruppi di coomologia e spazi di Stein*. Boll. Un. Mat. Ital. 18 B **5** (1981), 649–662.

[3] Carleson L.: *Interpolation by bounded analytic functions and the corona problem*, Ann. of Math. (2) **76** (1962), 547–559.

[4] Coeuré G.; Loeb J.J.: *A counterexample to the Serre problem with a Bounded Domain of* $\mathbb{C}^2$ *as Fiber*, Ann. Math. Second Series, **122**, No. 2 (1985), 329–334.

[5] Colţoiu M.: *On Barth's conjecture concerning* $H^{n-1}(\mathbb{P}^n \setminus A, \mathcal{F})$, Nagoya Math. J. **45** (1997), 99–123.

[6] Colţoiu M.: *q-convexity. A survey.* Complex analysis and geometry XII, Pitman Research Notes in Math. vol. **366**, 83–93.

[7] Colţoiu M.; Diederich K.: *On Levi's problem on complex spaces and envelopes of holomorphy*, Math. Ann. **316** (2000), no. 1, 185–199.

[8] Demailly J.-P.: *Un exemple de fibré holomorphe non de Stein à fibre* $\mathcal{C}^2$ *ayant pour base le disque ou le plan*. Invent. Math. **48** (1978), no.3, 293–302.

[9] Diederich K.; Fornæss J.E.: *Smoothing q-convex functions and vanishing theorems*, Invent. Math. **82** (1985), 291–305.

[10] Fornæss J.E.; Narasimhan R.: *The Levi problem on Complex Spaces with Singularities*, Math. Ann. **248** (1980), 47–72.

[11] Fornæss J.E.; Sibony N.: *Smooth pseudoconvex domains in $\mathbb{C}^2$ for which the corona theorem and L^p estimates for $\overline{\partial}$ fail*, Complex Analysis and Geometry, 209–222, Univ. Ser. Math., Plenum, New York, (1993).

[12] Garnett J.B.; Jones P.W.: *The corona problem for Denjoy domains*, Acta Math. **155** (1985), 27–40.

[13] Grauert H.; Lieb I.: *Das Ramirezsche Integral und die Lösung der Gleichung $\overline{\partial} f = \alpha$ im Bereich der beschränkten Formen*, Rice Univ. Studies **56** no.2, 29–50 (1971).

[14] Greene R.E.; Wu H.H.: *Embedding of open riemannian manifolds by harmonic functions*, Ann. Inst. Fourier (Grenoble) **25** (1975) 215–235.

[15] Henkin G.M.; Sergeev A.G.: *Uniform estimates of the solutions of the $\overline{\partial}$-equation in pseudoconvex polyherdra*. Mat. Sb. (N.S.) **112 (154)** (1980), no.4 (8), 522–567.

[16] Krantz S.G.: *The Corona Problem with two pieces of data*. Proc. Amer. Math. Soc. **138**, no.10, (2010), 3651–3655.

[17] Matsumoto K.: *On the cohomological completeness of q-complete domains with corners*, Nagoya Math. J. **168** (2002), 105–112.

[18] Pflug P.; Zwonek W.: *The Serre problem with Reinhardt fibers*. (English, French summary) Ann. Inst. Fourier (Grenoble) **54** (2004), no.1, 129–146.

[19] Preda O.: *Locally Stein open subsets in Normal Stein spaces*, J. Geom. Anal. **25** (2015), 2759–2766.

[20] Preda O.: *On the intersection of $(n-1)$-complete open subsets with $\mathcal{C}^2$ boundary in $\mathbb{C}^n$*, preprint.

[21] Preda O.: *The Corona problem with restrictions on the relative position of sublevel sets*, Arch. Math. **105** (2015), 563–569.

[22] Ramírez de Arellano E.: *Ein Divisionsproblem und Randintegraldarstellungen in der komplexen Analysis*, Math. Ann. **184** (1969/1970) 172–187.

[23] Sibony N.: *Problème de la couronne pour des domaines pseudoconvexes à bord lisse*, Ann. of Math. (2) **126** (1987), 675–682.

[24] Siu Y.-T.: *Holomorphic Fiber Bundles whose Fibers are Bounded Stein Domains with Zero First Betti Number*, Math. Ann. **219** (1976), 171–192.

[25] Skoda H.: *Application des techniques L^2 à la théorie des idéaux d'une algébre de fonctions holomorphes avec poids*. Ann. Sci. École Norm. Sup. (4) **5** (1972), 545–579.

[26] Skoda H.: *Fibrés holomorphes à base et à fibre de Stein*. Invent. Math. **43** (1977), no.2, 97–107.

[27] Zaffran D.: *Serre problem and Inoue-Hirzebruch surfaces*, Math. Ann. **319** (2001), 395–420.