SOME NUMERICAL ASPECTS IN MODELING THE LONGITUDINAL IMPACT OF TWO SHAPE MEMORY ALLOY BARS

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Lucrarea investighează aspecte dinamice ale transformărilor de fază în corpuri solide. Considerăm un model explicit termo-elastic unidimensional nemonoton capabil să descrie comportamantul termo-mecanic al unui aliaj cu memoria formei. Este analizat impactul longitudinal a doua bare ce pot suferi o transformare de fază care generează propagarea atât a undelor de şoc elastice cât şi a frontierelor intre diferitele faze solide ale materialului. Prezentam cateva rezultate numerice preliminare legate de caracterul exotermic/endotermic al transformării de fază directe/inverse, respectiv austenitică-martensitică.

This paper investigates dynamic aspects of solid-solid phase transformation. We consider an explicit one-dimensional non-monotone thermo-elastic model able to describe the thermo-mechanical response of a shape memory alloy. The longitudinal impact of two phase transforming bars that give rise to both adiabatic shock waves and propagating phase boundaries is analyzed. We present some preliminary numerical results related with the exothermic/endothermic character of the direct/inverse austenitic-martensitic phase transformation.

Keywords: bar impact, shock waves, shape memory alloys, thermo-mechanics.

1. Introduction

In a recent paper (see [1]) longitudinal impact experiments of thin bars have been proposed as an effective mean for understanding the kinetics of stress-induced phase transformations in shape memory alloys (SMA). This problem has been investigated in a one-dimensional and isothermal setting and has provided important insight into the wave structure. Since the influence of thermal effects on the rate of phase transformation, size and shape of pseudo-elastic hysteresis at higher strain rates is very important we have taken into account in [2] their influence on the wave propagation and nucleation of phases. In this paper we present some preliminary numerical results related with the exothermic/endothermic character of the direct/inverse austenitic-martensitic phase transformations in

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impact experiments. A complete analysis of this problem will be done in a forthcoming paper.

2. Adiabatic thermomechanical bar theory

We consider here the longitudinal adiabatic motions of thin cylindrical bars by taking into account simultaneously inertia and heat release. The bar is supposed to be thermally isolated and, moreover, the shocks and phase boundaries are treated as adiabatic, i.e. the heat conductivity can be neglected. The balance laws of momentum, mass and energy in a 1-D Lagrangian description are

$$\rho\frac{\partial v}{\partial t}-\frac{\partial \sigma}{\partial X}=0,\qquad \frac{\partial \varepsilon}{\partial t}-\frac{\partial v}{\partial X}=0,\qquad \rho\frac{\partial e}{\partial t}-\sigma\frac{\partial \varepsilon}{\partial t}=0, \tag{1}$$

where v, σ, ε, e and ρ are the particle velocity, the stress, the strain, the specific internal energy and constant mass density at a reference point X and time t, respectively.

Across the discontinuities, the system (1) must be supplemented by the Rankine-Hugoniot jump conditions and the dissipation inequality

$$\rho\dot{S}[v]+[\sigma]=0,\ \ \dot{S}[\varepsilon]+[v]=0,\ \ \rho\dot{S}[e+\frac{1}{2}v^2]+[\sigma v]=0,\ \ -\rho\dot{S}[\eta]\geq 0, \tag{2}$$

where $\eta = \eta(X, t)$ is the specific entropy and $\dot{S}$ is the Lagrangian velocity of the discontinuity. Here for the jump we use the standard notation $[.]=(.)_+-(.)_-$ and "+" corresponds to the state at the right of the discontinuity while "-" to the state at the left of the discontinuity.

3. Constitutive assumptions - three phase materials

We consider a material which exists in a high-temperature phase austenite A and has two variants M− and M+ of a low-temperature martensite. One variant is obtained in compression tests and the other one in tension tests, respectively. Starting with the paper [2] it is now a common use to model materials undergoing solid-solid phase transitions by using non-monotone, up-down-up stress-strain relations for certain intervals of temperatures. We consider here such an explicit simple stress-strain-temperature relation $\sigma=\sigma_{eq}(\varepsilon,\theta)$ derived from physical considerations on the behavior of shape memory alloys in quasi-static tests. It is inspired by the model proposed in [4] and used in [5] as an equilibrium relation for the following Maxwellian rate-type constitutive equation

$$\frac{\partial \sigma}{\partial t}-E\frac{\partial \varepsilon}{\partial X}=-\frac{E}{\mu}(\sigma-\sigma_{eq}(\varepsilon,\theta)) \tag{3}$$

where E=const.>0 is the dynamic Young modulus and μ=const.>0 is a Newtonian viscosity coefficient. Eq. (3) should be seen as a rate-type approach of the non-monotone thermo-elasticity when $\mu \to 0$. These two constitutive phenomenological models are used to describe the thermomechanical behavior of a polycristalline, near-equiatomic NiTi alloy. Material constants used for both models have been established in agreement with the laboratory experiments reported in [6].

4. Numerical results

This paper is concerned with the use of numerical simulations to define a proce-dure to characterize the material response of shape memory alloys in general, and of NiTi in particular, by using impact experiments. The proposed method is based on performing longitudinal impact experiments, at room temperature, of two phase transforming bars (see Fig.1). The first one, called flyer, of length *L*, moves with a known velocity V_0 and strikes the second one, called target, of length *l*, which is at rest. We choose in our numerical experiments L = 3l = 30mm.

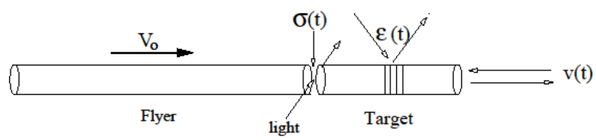
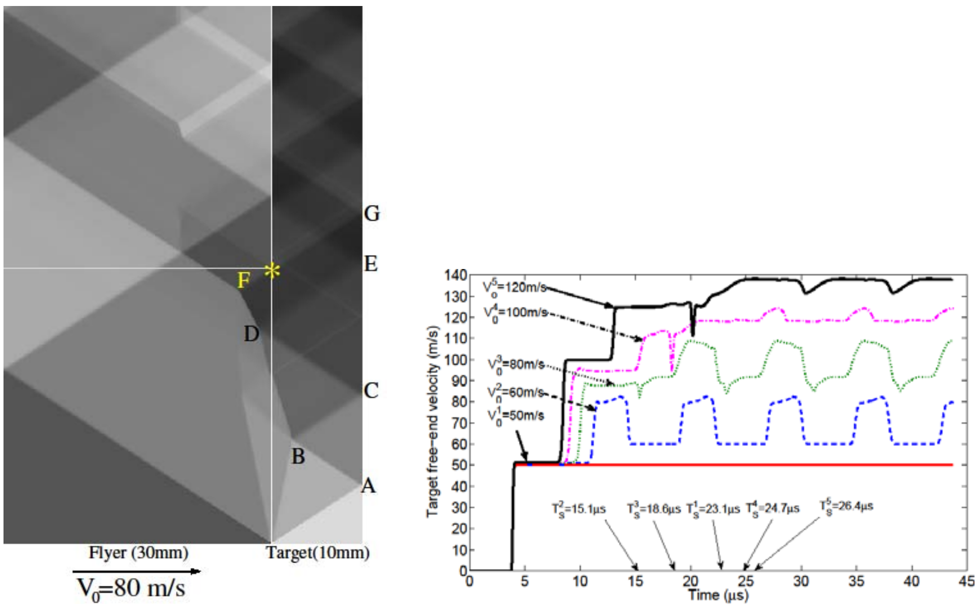


Fig. 1 Longitudinal impact of thin bars.

After impact the two bars remain in contact and move together until a time t_S called time of separation. This time corresponds to the moment when the first tensile wave arrives at the contact point.

In this simple dynamic laboratory test one can measure: the time of separation between bars, the particle velocity at the free-end of the target by interferometry, the variation in time of the strain at various cross-sections by using diffraction gratings, the stress history at the impacted end by piezoelectric wafers. The idea is to increase in a systematic manner the impact velocity V_0 and to try to correlate this input data with the above mentioned measurable quantities. In our numerical experiments we consider the material in the range of its pseudo-elastic behavior. Therefore the transformed zone induced by impact has to disappear after the separation of the bars.

One shows that if the impact velocity V_0 is less than a critical velocity V_{cr}=51 m/s, then the contact between the two bars is elastic. If this value is overcome then a transformed region, bounded by two propagating left and right phase boundaries, appears near the impact point. If the impact velocity is increased the speed of the phase boundary increases and the transformed zone extends as it is illustrated in a time-distance representation in Fig. 2a). This figure also gives an overview on the way elastic shock waves and phase boundaries propagate, are reflected at the free ends of the two bars and interact each other being reflected and/or transmitted.



a) b)

Fig.2 a) Velocity distribution in the bars after impact. b) Velocity history at the target free-end for different impact velocities.

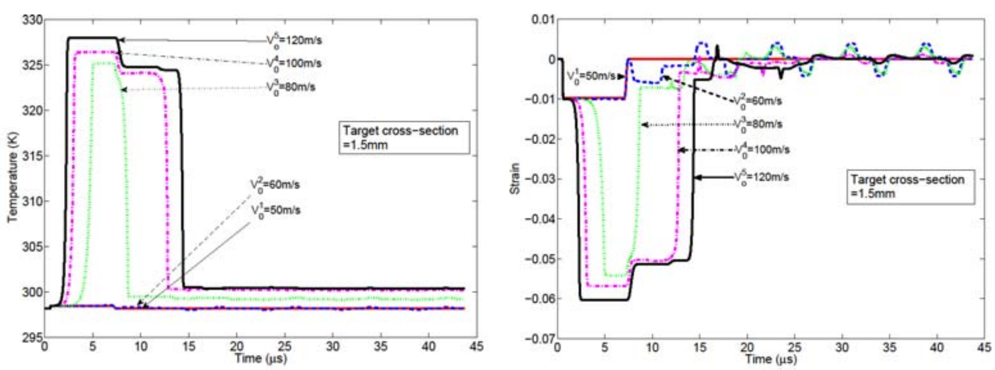


Fig. 3 Temperature and strain history for different impact velocities at the target cross-section equal to 1.5 mm from the impact point.

The presence of a phase transformation near the impact point can be detected by recording the changes in the velocity profile at the rear end of the target as in Fig. 2b). If the impact is elastic one records only one step increase of the velocity corresponding to the arrival of the elastic pulse. If a transformation is generated the adiabatic shock wave reflected by the phase boundary propagating in the target induces a new significant step like increase of the target free-end velocity.

Fig. 3 puts into evidence the exothermic/endothermic character of the direct/inverse austenitic-martensitic phase transformation. Indeed, one can observe, according to the numerical predictions of the Maxwellian rate-type model (3), how the temperature of the transformed zone increases and afterwards decreases when the transformed zone disappears.

4. Conclusions

In this paper we use the Maxwellian rate-type model (3) as a viscous regularization of the thermo-elastic model $\sigma = \sigma_{eq}(\varepsilon, \theta)$. This regularized framework is successful in removing deficiencies of the classical thermo-elastic model as the ill-posedness of initial and boundary values problems of the corresponding adiabatic PDEqs system. The longitudinal impact of phase transforming bars are an effective mean to determine the speed of propagation of phase boundaries in phase transforming materials. Indeed, the speed of propagation of phase boundaries can be determined in an indirect way by measuring the particle velocity at the free-end of the target bar. It is known that in phase transitions of shape memory alloys the heat is released when the material transforms from the low strain phase to the high strain phase and is absorbed in the reverse transformation. Through numerical experiments we put here into evidence the variation of the temperature in the transformed material and implicitly its influence on the wave propagation during the impact of the two bars.

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