INTERNATIONAL CONFERENCE IN ALGEBRAIC GEOMETRY

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TALK ABSTRACTS.

Lucian Bădescu. *A connecteness theorem for products of weighted projective spaces.*

Abstract: Let P be a product of weighted projective spaces, and let $f : X \to P \times P$ be a morphism from a projective irreducible variety X. We prove a connectedness theorem a la Fulton-Hansen. In other words, we prove that, under certain conditions,$f^{-1}(\Delta)$ is connected, where Δ is the diagonal of $P \times P$. Some applications are also given. This is work in collaboration with Flavia Repetto.

Fabrizio Catanese. *Moduli spaces of surfaces: deformation-differentiable-symplectic types.*

Abstract: Bidouble covers of quadrics, especially the so-called *abc*-surfaces, gave the strongest counter-examples to the def=diff conjecture of Friedman and Morgan, giving simply connected algebraic surfaces which are diffeomorphic (through a diffeomorphism carrying the canonical class to the canonical class) but not deformation equivalent. The final step is a long chain of arguments, ranging from deformation theory to the theory of smoothings of singularities, was done in a joint work with Bronek Wajnryb, using an explicit description of the mapping class group factorizations of certain Lefschetz fibrations in order to assert the diffeomorphism of *abc*-surfaces with the same b and the same $(a + c)$. A very difficult open question is whether diffeomorphic abc surfaces, endowed with their canonical symplectic structure, are symplectomorphic to each other. Work in progress with Michael Loenne and Bronek Wajnryb shows how to distinguish irreducible components of these moduli spaces via invariants of the braid monodromy factorization of the branch curves, realizing a first step in the direction of a program set uo manyy years ago by Boris Moishezon. This result leaves however open the question of symplectomorphism of the abc-surfaces.

Ciro Ciliberto. *Coalescence of fat points and applications.*

Abstract: In this talk, I will explain how the "blowup and twist" technique introduced by R. Miranda and myself for attacking the Segre-Harbourne-Gimigliano-Hirschowitz conjecture, can be exploited in order to analyse schemes arising from coalescence of fat points.

Alexandru Dimca. *On isotropic subspace theorems.*

Abstract: The isotropic subspace theorem is a refinement of Castelnuovo-DeFranchis Lemma. It was obtained in the projective case by F. Catanese and in the quasi-projective case by I. Bauer. The same Lemma was used by A. Neauville in relation to the study of characteristic varieties of rank one local systems. In the talk we explain a new relation between isotropic subspace theorems and characteristic

varieties, which improves the information one has in the class of quasi-projective 1-formal varieties. As an example beyond this class we will discuss the configuration spaces on n distinct labeled points on an elliptic curve.

Jean-Marc Drézet. *Geometry of multiple curves.*

Abstract: A *primitive multiple curve* is an algebraic curve X that can be locally embedded in a smooth surface, and such that the associated reduced curve X_red is smooth. Such curves have been defined and studied by C. Bănică and O. Forster in 1984. The complete classification of double primitive curves (called *ribbons*) has been done by D. Bayer and D. Eisenbud in 1995. In this talk we describe the classification of primitive multiple curves of any multiplicity, and give an overview of the study of coherent sheaves and their moduli on these curves.

Gavril Farkas. *The birational geometry of moduli spaces of curves with level structure.*

Abstract: Moduli of curves with level structure provide an interesting correspondence between the moduli spaces of curves and abelian varieties respectively. Using Koszul-theoretic methods we prove that the moduli space $\mathcal{R}_g$ of Prym varieties of dimension $g-1$ is of general type for $g > 13$, and that the moduli space S_g of spin curves is of general type for $g > 8$. In contrast that S_g has negative Kodaira dimension for $g < 8$.

Hubert Flenner. $\mathbf{C}_+-$ *and* $\mathbf{C}^*$*–actions on Gizatullin surfaces.*

Abstract: This is a report on a joint work with S. Kaliman (Florida State U.) and M. Zaidenberg (U. Grenoble). A Gizatullin surface is a normal affine surface V over $\mathbf{C}$ which can be completetd by a zigzag; that is, by a linear chain of smooth rational curves. We deal with the question of uniqueness of $\mathbf{C}^*$–actions and $\mathbf{A}^1$-fibrations on such surfaces up to automorphisms. The latter fibrations are in one to one correspondence with $\mathbf{C}_+$–actions on V considered up to "speed change". Non-Gizatullin surfaces are known to admit at most one $\mathbf{A}^1$-fibration $V \to S$ up to an isomorphism of the base S. Moreover an effective $\mathbf{C}^*$-action on them, if it does exist, is unique up to conjugation and inversion is unique up to conjugation and inversion $t \mapsto t^{-1}$ of $\mathbf{C}^*$. The main results are:

(1) We can classify all smooth Gizatullin surfaces for which there are several conjugacy classes of $\mathbf{C}^*$-action. It turns even out that there are surfaces with 1- and 2-parameter families of $\mathbf{C}^*$-actions which are pairwise non-conjugate.
(2) We obtain a criterion as to when $\mathbf{A}^1$-fibrations of Gizatullin surfaces are unique. For instance, by this result we can exhibit a large subclass of Gizatullin $\mathbf{C}^*$-surfaces for which there are at most two conjugacy classes of $\mathbf{A}^1$-fibrations over $\mathbf{A}^1$.

Anthony Iarrobino. *An Algebra of Commuting Nilpotent Matrices.*

Abstract: Let $Mat_n(K)$ denote the ring of $n \times n$ matrices over a field K. Fix a nilpotent $n \times n$ matrix B of Jordan partition P, and consider the centralizer $\mathcal{C}_B$ of B, and its subvariety $\mathcal{N}_B$ of nilpotent matrices. Denote by $N^2(n, K)$ the variety of commuting pairs of nilpotent matrices. We describe recent work on both these varieties, and the connections with previous work by J. Briançon et al on the fibre $H^{[n]}$ of the punctual Hilbert scheme $Hilb^n(P^2)$ of the plane over a point $p \in P^2$.

R. Basili defined a maximal nilpotent subalgebra $\mathcal{U} = \mathcal{U}_B$ of $\mathcal{N}_B$. We describe an involution on $\mathcal{C}_B$, and give bases for the quotients $\mathcal{U}^i/\mathcal{U}^{i+1}$ (work joint with R. Basili).

Atanas Iliev. *Birational geometry and period map for Fano threefolds of degree 10.*

Abstract: The Fano threefold $X(10)$ of degree 10 is the smooth complete intersection of the Grassmannian $G(2,5)$ with two hyperplanes and a quadric. Just like for the quartic double solid $Y(2)$, the intermediate Jacobian of $X(10)$ is 10-dimensional, in addition the Fano surfaces of $X(10)$ and $Y(2)$ have the same invariants. Nevertheless, the general $X(10)$ is not birational to a quartic double solid $Y(2)$. Moreover, while for $Y(2)$ there is a Torelli theorem, the Torelli theorem does not hold for $X(10)$, since it has 22 moduli, while the period map for $X(10)$ has a 20-dimensional image, With the help of the birational geometry of $X(10)$, we are able to describe the general fiber of the period map for $X(10)$, which turns out to be the union of two surfaces - the quotient Fano surface of $X(10)$ and its dual surface. At the end we discuss some open problems. This talk is based on a common work with O. Debarre and L. Manivel.

Herbert Lange. *Prym-Tyurin varieties via Hecke algebras.*

Abstract: (joint work with A. Carocca, R. Rodriguez, A. Rojas) Let G denote a finite group and $\pi : Z \to Y$ a Galois covering of smooth projective curves with Galois groups G. For every subgroup H of G there is a canonical action of the corresponding Hecke algebra $\mathbb{Q}[H\backslash G/H]$ on the jacobian of the curve $X = Z/H$. To each rational irreducible representation $\mathcal{W}$ of G we associate an idempotent in the Hecke algebra, which induces a correspondence of the curve X and thus an abelian subvariety P of the Jacobian JX. We give sufficient conditions on $\mathcal{W}$, H and the action of G on Z, which imply P to be a Prym-Tyurin variety. We obtain many new families of Prym-Tyurin varieties of arbitrary exponent in this way.

Antonio Lanteri. *Revisiting classification by sectional genus in the setting of ample vector bundle.*

Abstract: Let X be a smooth complex projective variety of dimension n and let F be an ample vector bundle of rank $n-1$ on X. The curve genus g of (X, F) is defined by $2g-2 = (K_X + c_1(F))c_{n-1}(F)$. Pairs (X, F) with low g are completely understood for $g < 2$ and partially for $g = 2$. As a preliminary step to understand pairs (X, F) with $g = 3$ we consider vector bundles $F = E \oplus H^{\oplus(n-r-1)}$, where H is an ample line bundle of rank r with a section vanishing on a smooth subvariety Z of X of the expected dimension. In this setting, a structure theorem for triplets (X, E, H) as above will be discussed under the assumption that the restricted line bundle H_Z is very ample and (Z, H_Z) is a projective manifold of sectional genus three (joint work with Maeda). The proof combines Ionescu's classification of projective varieties of projective varieties of low sectional genus with results of adjunction theory for ample vector bundles.

Laurentiu Maxim. *Hirzebruch-type invariants of complex algebraic varieties.*

Abstract: In the first part of the talk I will present joint work with S. Cappell and J. Shaneson on the behavior of Hirzebruch-type invariants (genera and characteristic classes) under proper morphisms of complex algebraic varieties. I will discuss formulae that relate globa invariants of a variety X to such invariants of singularities of maps defined on X. Such formulae severely constrain, both topologically and analytically, the singularities of proper morphisms, even between smooth varieties. The second part of the talk is devoted to work, joint also with A. Libgober and J. Schuermann, on the study of mondromy contributions to the computation of

Hirzebruch-type invariants of fiber bundles. I will describe Hodge-theoretic formulae of Atiyah-Meyer type that measure the deviation from multiplicativity of these Hirzebruch invariants.

Massimiliano Mella. *Equivalent birational embeddings.*

Abstract: Let us consider a rational variety $X^r \subset \mathbb{P}^n$. Then there exists a birational map $\phi : X \dashrightarrow \mathbb{P}^r$. The simplest embedding of $\mathbb{P}^r$, as a projective variety is the linear one. It is quite natural to ask whether the map ϕ can be extended to a birational map $\Phi : \mathbb{P}^n \dashrightarrow \mathbb{P}^n$ such that $\Phi(X)$ is linear. It is not difficult o imagine that when XX is a divisor in $\mathbb{P}^n$ this is not possible in general. What is really surprising is that this is the only case in which the answer is negative.

Anca Mustaţă. *Weighted stable map spaces and their cohomology.*

Abstract: In joint work with Andrei Mustaţă we study families of compactifications of the spaces of maps from smooth curves to projective varieties, generalizing Kontsevich and Manin's moduli spaces of stable maps. These compactifications correspond to families of stability conditions placed on maps. The relations among these stability conditions lead to insights into the geometry and cohomology of the stable map spaces.

Mircea Mustaţă. *Sequences of log canonical thresholds.*

Abstract: The log canonical threshold of a function is an invariant that measures which powers of the function are locally integrable. This is an invariant that appears in many contexts, and that found many applications in birational geometry. I will discuss results and conjectures dealing with limit points of their log canonical thresholds. This is joint work with Tommaso de Fernex.

Jan Nagel. *On the motive of a conic bundle over a surface.*

Abstract: We give a decomposition of the Chow motive of a conic bundle over a smooth surface. Our result implies (up to isogeny) a theorem of Beauville, which relates the Chow group of codimension two cycles algebraically equivalent to zero on the conic bundle to the Prym variety of the associated double covering of the discriminant curve. (Joint work with M. Saito.)

Daniel Naie. *Jumping numbers of a unibranch curve on a smooth surface.*

Abstract: A formula for the jumping numbers of a curve unibranch at a singular point is established. The jumping numbers are expressed in terms of the Enriques diagram of the log resolution of the singularity, or equivalently in terms of the canonical set of generators of the semigroup of the curve at the singular point.

Giorgio Ottaviani. *Invariants regarding Waring problem for polynomials.*

Abstract: The Waring problem for polynomials asks to express a homogeneous polynomial as a sum of powers of linear forms. It has a geometrical intrepretation with the higher secants of Veronese varieties and it can be applied to polynomial interpolation. We will describe some invariants occurring in this setting, and in particular the condition to express a cubic polynomial in five variables as the sum of seven cubes.

Ştefan Papadima. *Twisted cohomology and Aomoto complexes of* 1*-formal groups.* (joint work with A. Dimca and A. Suciu)

Abstract: The groups in the title are generalizations of fundamental groups of compact Kähler manifolds. For the latter, there are known connections between

twisted cohomology with rank 1 complex coefficients, and isotropic subspaces coming from the cup-product map in low degrees. I will explain how the above result extends to 1-formal groups, and twisted coefficients of arbitrary rank.

Giuseppe Pareschi. *Generic vanishing, Fourier-Mukai trasform, and syzygies.*

Abstract: Everybody knows the importance of the vanishing of higher cohomology groups of coherent sheaves. On irregular varieties there is a natural weakening of such property, namely generic vanishing. This means that, up to twisting with a general topologically trivial line bundle, the higher cohonology vanishes. In this talk I will consider a quantitative measure of generic vanishing – the generic vanishing index – and I will describe its relation with the Fourier-Mukai trasform of the coherent sheaf in question. A somewhat surprising connection with local commutative algebra appears. As an application, one obtains a generalization to arbitrary dimension of the classical Castelnuovo-de Franchis inequality. Joint work with Mihnea Popa.

Mihnea Popa. *BGG correspondence and the cohomology algebra of irregular Kähler manifolds.*

Abstract: I will explain how to exploit, via the Bernstein-Gelfand-Gelfand (BGG) correspondence and Generic Vanishing theprems, the fact that the cohomology algebra of the structure sheaf of an irregular Käher manifold is a module over the exterior algebra. The main result will be a calculation of the regularity of this module for varieties with generically finite Albanese map. The BGG correspondence will then provide linear resolutions of vector bundles on projective space, which in turn yield by vector bundle theoretic methods bounds for the Hodge numbers of irregular varieties. This describes joint projects in progress with R. Lazarsfeld and G. Pareschi.

Kristian Ranestad. *Toric polar Cremona transformations.*

Abstract: I shall report on common work with Frank Sottile and Hans-Christian Graf von Bothmer on the problem: For which homogeneous polynomials $F(x, y, z)$ does $x : y : z \to xF_x : yF_y : zF_z$ where F_x is the partial of F w.r.t. x etc, define a Cremona transformation ?

Francesco Russo. *The Matrioska principle for varieties uniruled by lines.*

Abstract: Some projective varieties with *extremal* geometrical properties (e.g. smooth secant defective varieties with the largest defect, smooth varieties whose dual is very small, particular homogeneous varieties etc, etc) are naturally uniruled by lines. In the smooth case the Hilbert scheme of lines passing through a general point of many of these varieties os also a smooth irreducible variety uniruled by lines, which usually has stronger geometrical properties of the same kind (smaller codimension, greater defect with respect to dimension, etc, etc).

This behaviour allows an inductive approach for the classification os most *extremal* varieties leading to a *matrioska of varieties uniruled by lines* (the smallest object with respect to codimension being the original variety) and hence to restrictions for the existence of these objects.

We shall illustrate these principles through a new geometrical approach to the classical problem of the possible projective extensions of smooth projective varieties uniruled by lines and through a strong Divisibility result relating the dimension and the defect of special secant defective varieties leading to significant applications.

Frank-Olaf Schreyer. *Betti numbers of graded modules.*

Abstract: It can be very difficult to analyze for a given system of polynomial equations qualitative properties, such as the geometry of the corresponding variety. The theory of syzygies offers a tool for looking at systems of equations, which might help to make their subtle properties visible. In a recent paper, Boij and Söderberg introduced a series of conjectures, which characterize all possible syzygy numbers of Cohen-Macaulay modules up to rational multiples. In this talk I report on the proof of these conjectures.

Alexandru Suciu. *Which Kähler groups are 3-manifold groups?.*

Abstract: Every finitely presented group G can be realized as the fundamental group of a smooth, compact, connected (orientable) 4-dimensional manifold. Requiring that G be the fundamental group of a Kähler manifold, or that of a 3-manifold, is very restrictive. A natural question, raised by Donaldson, Goldman, and Reznikov in the 1990s, is then: What if both conditions are required to hold? We give a complete answer, as follows: if G can be realized as both the fundamental group of a closed 3-manifold and of a compact Kähler manifold, then G must be finite, and thus belongs to the well-known list of finite subgroups of $O(4)$. This is joint work with Alexandru Dimca (arXiv:0709.4350, to appear in JEMS).

Claire Voisin. *Potential density of rational points for the variety of lines of a cubic fourfold.*

Abstract: This is joint work with E. Amerik. We show that for many cubic fourfolds defined over a number field K, there is a number field L such that the L-points of the variety of lines $F(X)$ are Zariski dense in $F(X)$. These varieties $F(X)$ have Picard number 1 (for $K3$ surfaces, there is no example with Picard number 1 known to satisfy the above density property, while it is conjectured to always hold).