Ph. D. Thesis
Abstract

Stochastic analysis and potentials - ergodicity and quasimartingales for Markov processes

Iulian Cîmpean

Supervisor : Lucian Beznea

Bucharest, 2016

This thesis is devoted to the study of global and local properties for Markov processes and their semigroups and resolvents, by using analytic and probabilistic potential theoretical tools: recurrence, transience, irreducibility, and ergodicity in L^p-spaces, existence of the invariant measures, quasimartingale and semimartingale functionals on Markov spaces. The framework is general enough to permit applications to infinite dimensional analysis via Dirichlet forms, to SPDEs on Hilbert spaces, or to measure-valued processes.

Besides some preliminaries collected in the first chapter, we distinguish four different central topics (briefly presented below) treated separately in four chapters.

In the first part we analyze the transience, recurrence, and irreducibility properties of general sub-Markovian resolvents of kernels and their duals, with respect to a fixed sub-invariant measure m. We give a unifying characterization of the invariant functions, revealing the fact that an L^p-integrable function is harmonic if and only if it is harmonic with respect to the weak dual resolvent. Our approach is based on potential theoretical techniques for resolvents in weak duality. We prove the equivalence between the m-irreducible recurrence of the resolvent and the extremality of m in the set of all invariant measures, and we apply this result to the extremality of Gibbs states. We also show that our results can be applied to non-symmetric Dirichlet forms, in general and in concrete situations. A second application is the extension of the so called *Fukushima ergodic theorem* for symmetric Dirichlet forms to the case of sub-Markovian resolvents of kernels.

Second, we give a new, two-step approach to prove existence of finite invariant measures for a given Markovian semigroup. First, we identify a convenient auxiliary measure and then we prove conditions equivalent to the existence of an invariant finite measure which is absolutely continuous with respect to it. As applications, we give a very short proof for the result of Lasota and Szarek on invariant measures and we obtain a unifying generalization of different versions for Harris' ergodic theorem which provides an answer to an open question of Tweedie. We show that for a nonlinear SPDE on a Gelfand triple, the strict coercivity condition is sufficient to guarantee the existence of a unique invariant probability measure for the associated semigroup, once it satisfies a Harnack type inequality. A corollary of the main result shows that any uniformly bounded semigroup on L^p possesses an invariant measure and we give some applications to sectorial perturbations of Dirichlet forms.

In the third part, for a fixed right process X we investigate those functions u for which $u(X)$ is a quasimartingale. The study relies on an analytic reformulation of the quasimartingale property for $u(X)$ in terms of a certain variation of u with respect to the transition function of the process. We prove that $u(X)$ is a quasimartingale if and only if u is the difference of two finite excessive functions. In particular, we show that the quasimartingale nature of u is preserved under killing, time change, or Bochner subordination. We provide sufficient conditions under which $u(X)$ is a quasimartingale, and finally, we extend to the case of semi-Dirichlet forms a semimartingale characterization of such functionals for symmetric Markov processes, due to Fukushima.

Eventually, by a direct approach based on Lusin's theorem, we prove the Bochner-Kolmogorov theorem on the existence of the limit of projective systems of second countable Hausdorff (non-metrizable) spaces with tight probabilities, such that the projection mappings are merely measurable functions. The motivation of revisiting this classical result comes from an application to the construction of the continuous time fragmentation processes and related branching processes.

The thesis consists mainly from the papers [BeCîRö 15], [BeCîRö 15a], [BeCî 16], and [BeCî 14].

In the sequel, we briefly detail the structure and main results of this work and we mention that chapters 2-5 correspond to the four main topics mentioned above.

In Chapter 1, *Preliminaries*, for the reader's convenience but also to fix the notations, we recall some more or less classical definitions and results which are intensively used throughout the forthcoming chapters: resolvent of kernels, excessive functions, duality, Markov processes and related potential theory, elements of Dirichlet forms. Here, in order to highlight several standard tools or techniques which have been employed in proving most of the main results of the present work, we tried to indicate at least the section where those specific results were involved.

The purpose of the second chapter, entitled *Ergodic properties for resolvents and applications*, is twofold: first, to clarify the connection between different definitions for transience, recurrence, and irreducibility, and unify various characterizations of these notions. Second, we want to analyze whether transience, recurrence, and irreducibility are stable when passing to the dual structure, i.e. the dual Markov process or the dual resolvent, respectively, with the underlying measure being a sub-invariant measure for the initial resolvent. On the way, we also obtain a number of new results on the subject, based on potential theoretical techniques.

Motivated by relevant examples arising mainly in infinite dimensional settings, we present here an approach to this subject in an L^p-context, for sub-Markovian resolvents. It turns out to be a unifying method, in particular, revealing applications to invariant and Gibbs measures.

Questions on recurrence, transience and irreducibility of Markov processes were treated in various frames and with specific tools, both from probabilistic and analytic view point: see [ChenFu 11], [Get 80], [Oshi 92], [Sturm 94], [Fu 07], [FuOsTa 11], and [MaUeWa 12] for continuous time processes, as well as [MeTw 93] and [Norr 97] for Markov chains, and the references therein.

The structure and main results of Chapter 2 are as follows.

In the first part of Section 2.2 we study different characterizations of transience, recurrence, and irreducibility of a sub-Markovian resolvent of kernels $\mathcal{U}$ on a Lusin measurable space E with respect to a σ-finite sub-invariant measure m. We emphasize that we do not assume any continuity of the resolvent and our proofs rely on the weak duality for the resolvent $\mathcal{U}$, and corresponding potential theoretical techniques, which is in contrast to the ones in [Fu 07] and [FuOsTa 11], where main ingredients are *Hopf's maximal inequality* and the continuity of the transition function. When $\mathcal{U}$

is the resolvent of a right process, we show that m-transience and m-irreducible recurrence are respectively equivalent with the transience and recurrence of the process in the stronger sense of [Get 80], outside some *m-inessential* set. This probabilistic counterpart was studied in [FuOsTa 11] for m-symmetric Hunt processes. Then, we give a characterization for invariant functions in $L^p(E, m)$, $1 \leq p \leq \infty$, unifying the approaches from stochastic processes, Dirichlet forms, positivity preserving semigroups, and ergodic theory. Our results also cover and extend the ones in [Schi 04], and we shall use them in Sections 2.3 and 2.4 to prove the equivalence of irreducibility and extremality of invariant measures, resp. extremality of Gibbs states. A second consequence of states that an element u from the kernel of the generator of an L^p-strongly continuous sub-Markovian resolvent of contractive operators also belongs to the kernel of the co-generator on L^p induced by weak duality.

In Section 2.3 we apply the results of the previous one to prove the equivalence of irreducible recurrence and ergodicity of a sub-Markovian resolvent of kernels with respect to a sub-invariant σ-finite measure, extending the so called *Fukushima ergodic theorem* for a (quasi)regular Dirichlet form; see [FuOsTa 11], Theorem 4.7.3 and [AlKoRö 97a], Theorem 4.6. The key ingredient states the strong convergence of an L^p-uniformly bounded resolvent family of continuous operators $(\alpha U_\alpha)_{\alpha>0}$ to the projection on the kernel of $\mathcal{I} - \beta U_\beta$, as α tends to 0, for one (hence for all) $\beta > 0$.

The central result of Section 2.4 states that the sub-Markovian resolvent of kernels $\mathcal{U}$ is m-recurrent and m-irreducible if and only if the measure m is extremal in the set of all invariant probability measures for $\mathcal{U}$. This extends results from [AlKoRö 97a], [AlKoRö 97b], and [DaZa 96], Section 3.1, concerning the ergodicity and extremality of invariant measures.

In Section 2.5 we apply the obtained results on transience, recurrence, irreducibility, and extremality of invariant measures to the context of (non-symmetric) Dirichlet forms. We show that under the strong sector condition, the recurrence of a Dirichlet form is determined by its symmetric part. As applications, we extend to the non-symmetric case two well known recurrence criteria developed for symmetric forms, one in terms of a sequence of elements from the domain of the form which is increasing to 1 and converging in energy to 0 (as in [Oshi 92] and [FuOsTa 11]), and the other one in terms of the volume growth of balls (cf. [Sturm 94]). A similar generalization is done for transience. We also give a characterization for the irreducibility of a Dirichlet form. It improves the one in [AlKoRö 97a], Proposition 2.3, where the forms are symmetric, recurrent, and given by a square field operator. We would like to point out another consequence, namely that both the recurrence and the irreducibility of a strongly sectorial (non-symmetric) Dirichlet form is equivalent to the respective property of its symmetric part. We illustrate this by a concrete example in infinite dimensions.

The main results of this last section are given in a subsection on the extremality of Gibbs states. Recall that in [AlKoRö 97a] the authors extend classical results of Holley and Stroock for the Ising model, proving that a Gibbs state is extremal if and only if the corresponding Dirichlet form is irreducible (or equivalently ergodic), for classes of lattice models with non-compact, but linear spin space. In particular, numerous examples of irreducible Dirichlet forms on infinite dimensional state space are obtained. For applications to more general models see [AlKoRö 97b]. The purpose is to recapture two of the main results in [AlKoRö 97a] as particular cases of our results and thus to place the problem in a broader context. The key point is a result which states that the space of Gibbs measures which are absolutely continuous with respect to a fixed Gibbs measure m coincide with the space of all $\mathcal{U}$-invariant probability measures which are absolutely continuous with respect to m; here, $\mathcal{U}$ is the resolvent of the Dirichlet form. The main result here is on the equivalence between the extremality of Gibbs states and irreducibility of the corresponding Dirichlet form.

In Chapter 3, entitled *The existence of invariant measures for Markovian semigroups*, we focus on invariant measures which are key objects in ergodic theory. More precisely, we deal with the question of existence of finite invariant measures for Markovian semigroups. This problem has been studied by many authors over the last decades, from various points of view; see e.g. the monographs [MeTw 93], [DaZa 96], and the references therein.

If the underlying space E is a Polish space, the semigroup is given by the transition probabilities of a Markov process and is Feller (i.e., it maps the space of bounded continuous real-valued functions on E into itself), then one can obtain the existence of an invariant measure by applying the result of [LaSz 06], provided that there is a compact subset of E which is infinitely often visited by the process. Although these hypotheses are verified in many examples, sometimes they are quite difficult or even impossible to check, especially if the state space is of infinite dimensions. Another technique to obtain invariant measures is to make use of Harris' theorem and its refined versions, cf. e.g. [MeTw 93], [MeTw 93b], [MeTw 93c], [MeTw 93d], [DoFoGu 09], and [Hai 10]. In contrast to the previously mentioned, these results involve non-topological assumptions such as the existence of *small* sets (in the sense which will be made precise in Subsection 3.2.2) that are infinitely often visited. This kind of test sets are encountered, provided the associated process is irreducible; see [MeTw 93], Theorem 5.2.2. Invariant measures have also been investigated from an analytic perspective, as in [BoRöZh 00] and [Hino 00], by working with strongly continuous Markovian semigroups on L^p, $1 < p < \infty$. Examples of this situation arise by considering sectorial perturbations of Dirichlet forms satisfying some functional inequalities (see Subsection 3.2.4 below).

The main purpose of Chapter 3 is to give a new approach to the existence of invariant measures for Markovian semigroups, consisting of two steps. First, we construct a convenient *auxiliary* measure m and then we give conditions on the pair (P_t, m) which characterize the existence of a non-zero integrable co-excessive function for $(P_t)_{t\geq 0}$, regarded as a semigroup on $L^\infty(m)$, which is equivalent to the existence of an invariant measure for $(P_t)_{t\geq 0}$, which is absolutely continuous with respect to m. Therefore, we call the procedure proposed above *the two-step approach*; see Subsection 3.1.2. We point out that our main results are entirely measure theoretic and also do not involve irreducibility properties of the semigroup.

Several applications are considered: In Subsection 3.2.1, although not in its full generality, we give a very short proof of the well known result of Lasota and Szarek [LaSz 06]. Here, the two-step approach gives an additional benefit because it particularly entails the absolute continuity of the obtained invariant measure with respect to the fixed auxiliary measure.

In Subsection 3.2.2 we unify various versions of Harris' theorem to a much more general one, which contains all of these as special cases. As a byproduct we give an answer to an open question mentioned by Tweedie [Tw 01].

In Subsection 3.3.3 we show that for a nonlinear SPDEs on a Gelfand triple $V \subset H \subset V^*$, under a Wang's Harnack type inequality, the strict coercivity condition with respect to the H-norm is sufficient to guarantee the existence of a unique invariant probability measure for the solution. This result improves the ones from [Liu 09] and [Wa 13] where the embedding $V \subset H$ must be compact and the strict coercivity is considered with respect to the stronger V-norm. We also consider a perturbation of a Markov kernel satisfying a combined Harnack-Lyapunov condition, for which the result of Tweedie can not be used, but for which our two-step approach works easily. We also discuss the applicability of Harris' result for this kind of perturbation. The last part of this subsection was written taking into account a kind remark of Martin Hairer.

In Subsection 3.4 we study the case of uniformly bounded C_0-semigroups on L^p, $p \geq 1$. Implementing our two-step approach we obtain new applications for semigroups coming from small perturbations of Dirichlet forms, generalizing [BoRöZh 00] and [Hino 98].

In Chapter 4, entitled *Semimartingale functionals associated to Markov processes*, we turn our attention to another central topic, and to do this let us consider a (right) Markov process $X = (\Omega, \mathcal{F}, \mathcal{F}_t, X_t, \mathbb{P}^x)$ with state space E. In the celebrated paper [ÇiJaPrSh 80], the authors prove that a real-valued function u on E has the property that $u(X)$ is a semimartingale for each $\mathbb{P}^x$ if and only if there exists a sequence of finely open sets $(E_n)_{n\geq1}$ such that $\bigcup_n E_n = E$, the exit times T_n of E_n tend to infinity a.s., and u is the difference of two 1-excessive functions on each E_n. This characterization was later approached by Fukushima in [Fu 99] from a Dirichlet forms theory perspective. More precisely, he showed that if X is associated with a symmetric Dirichlet form $(\mathcal{E}, \mathcal{F})$ and $u \in \mathcal{F}$, then $u(X)$ is a semimartingale if and only if there exist a nest $(F_n)_{n\geq1}$ and constants $(c_n)_{n\geq1}$ such that for each $n \geq 1$

$$|\mathcal{E}(u, v)| \leq c_n \|v\|_\infty \text{ for all } v \in \mathcal{F}_{b,F_n}. \tag{1}$$

The idea of Fukushima in order to prove the sufficiency of inequality (1) was to assume first that $\mathcal{E}$ is a regular Dirichlet form so that, by Riesz representation, one has $\mathcal{E}(u, v) = \nu(v)$ for some Radon measure ν on E. The next step was to show that ν is a smooth measure, which means that the CAF from Fukushima decomposition is of bounded variation, hence $u(X)$ is a semimartingale. The extension to quasi-regular symmetric Dirichlet forms was achieved via the so called "transfer method". This result was then used by the author in order to develop a deep stochastic counterpart of BV functions in both finite and infinite dimensions; beside the above mentioned paper, we refer the reader also to [Fu 00] and the references therein. As a matter of fact, the approach using Dirichlet forms dates back to the work of Bass and Hsu in [BaHs 90] where they showed that the reflected Brownian motion in a Lipschitz domain is a semimartingale, result which was later extended to (strong) Caccioppoli sets in [ChFiWi 93], where the authors investigate the quasimartingale structure of the process. It is worth to mention that in [ChFiWi 93] the authors consider quasimartingales only on finite intervals and not on the entire positive semi-axis, as we do. Although it might seem a small difference, it is in fact the key point which makes our hole study achievable and, to the best of our knowledge, new.

The aim of Chapter 4 is twofold: first, we investigate those real-valued functions u on E for which $u(X)$ is a quasimartingale, and second, by means of semi-Dirichlet forms, we study those functions u for which $u(X)$ is a semimartingale by looking at their local quasimartingale structure. We briefly present below the structure and the main results of this chapter:

In Section 4.1 we show that the quasimartingale property of $u(X)$ may be reformulated in terms of the variation

$$V(u) := \sup_{\tau}\{\sum_{i=1}^{n} P_{t_{i-1}}|u - P_{t_i - t_{i-1}}u| + P_{t_n}|u|\}$$

of u w.r.t. the semigroup $(P_t)_{t\geq 0}$ of the process, which allows us to perform the study from a purely analytic point of view. The central results are mainly saying that $\{x \in E \ : \ u(X) \text{ is a quasimartingale w.r.t. } \mathbb{P}^x\} = \{V(u) < \infty\}$, and that $u(X)$ is a quasimartingale (which by convention means for all $\mathbb{P}^x, x \in E$) if and only if u may be decomposed as the difference of two finite excessive functions. In particular, if the process is irreducible and $(e^{-\alpha t}u(X_t))_{t\geq 0}$ is a $\mathbb{P}^{x_0}$-quasimartingale for one $x_0 \in E$, then it is a $\mathbb{P}^x$-quasimartingale for all $x \in E$. A Riesz type decomposition and some remarks on the space of differences of excessive functions are discussed in the end of the section.

In Section 4.2 we show that the quasimartingale property of functions is preserved under killing, time change, and Bochner subordination. In addition, we show that for a multiplicative functional M with permanent points E_M, $(e^{-\alpha t}M_t u(X_t))_t$ is a quasimartingale if and only if $(e^{-\alpha t}u|_{E_M}(X^M))_t$ is a quasimartingale, where X^M stands for the killed process by M. We also show that if $(e^{-\alpha t}u(X_t))_t$ is a quasimartingale, then so is the process $(e^{-\alpha \tau_t}u(Y_t))_t$, where τ is the inverse of an additive functional of X and Y denotes the corresponding time change process.

In Section 4.3 we provide tractable conditions for u such that $(e^{-\alpha t}u(X_t))_t$ is a quasimartingale. We distinguish two ways of considering such conditions, which we treat separately: the first one involves the resolvent $\mathcal{U} = (U_\alpha)_\alpha$ of the process, while the second approach is performed in an $L^p(\mu)$-context, where μ is a σ-finite sub-invariant measure. On brief, the point is that estimates of the type $U_\alpha(|P_t u - u|) \lesssim t$ in the case of the first approach, and of the type $\mu(|P_t u - u|f) \lesssim t\|f\|_\infty$ in the L^p-context, are sufficient to guarantee quasimartingale properties for $u(X)$. We also present a condition in terms of the dual generator on L^p-spaces.

In Section 4.4 we look at quasimartingale and semimartingale functionals from the Dirichlet form theory point of view. More precisely, if $(\mathcal{E}, \mathcal{F})$ is a (non-symmetric) Dirichlet form, then for an element $u \in \mathcal{F}$, an inequality of the type

$$|\mathcal{E}(u, v)| \leq c\|v\|_\infty \text{ for all } v \in \mathcal{F}_b \tag{2}$$

ensures that $(e^{-\alpha t}u(X_t))_t$ is a quasimartingale. As a matter of fact, we show that this is true under a more general situation, when $\|v\|_\infty$ in (2) is replaced by $\|v\|_\infty + \|v\|_{L^2(\mu)}$. Then we extend the previously mentioned semimartingale characterization due to Fukushima, to non-symmetric Dirichlet forms. Furthermore, we consider the situation when u is not necessarily in $\mathcal{F}$ (e.g. $u \in \mathcal{F}_{\mathrm{loc}}$), under the additional hypothesis that the form has the local property. At this point we would like to emphasize that in contrast with previous work, in order to prove the sufficiency of conditions (1) or (2) we do not use Fukushima decomposition or Revuz correspondence. Instead, we employ heavily the results of the previous sections, and as a matter of fact, this approach enables us to extend our results to semi-Dirichlet forms without further conditions. The section ends with a few remarks concerning situations when it is sufficient to check inequalities (1) or (2) for v belonging to a proper subspace of $\mathcal{F}$, like cores or special standard cores.

The last part of the thesis (Chapter 5, entitled *Bochner-Kolmogorov Theorem*) is devoted to the classical Bochner-Kolmogorov theorem. We prove a result on the existence of the limit of a projective system that consists of second countable Hausdorff topological spaces, not necessarily metrizable, with tight probabilities. The above topological hypothesis are automatically satisfied if the spaces are Lusin (or more general, Radon) topological spaces and it is straightforward to extend the result to Lusin measurable spaces.

Our motivation is given by the applications in [BeDeLu 15] and [BeDeLu 15a] of such a result to the construction of a branching process associated to a fragmentation one and respectively to develop a probabilistic model of the fragmentation phase of an avalanche.

Recall that the classical Kolmogorov extension theorem guarantees that a compatible collection of finite-dimensional distributions will define a stochastic process. Bochner (see [Boch 55]) considered the abstract situation of projective systems of Hausdorff topological measure spaces and proved the existence of the limit, provided that the measures can be approximated by compacts and replacing the canonical projections by continuous mappings.

Recent applications of Bochner-Kolmogorov theorem in proving non-trivial results about spatial systems, such as the existence of Gibbs states (see for example [KoPaRö 12] and [Pres 05]), require rather measure theoretical assumptions than topological ones. A significant improvement which is convenient to these applications was obtained by Parthasaraty (see [Parth 67], Chapter V, Theorem 3.2). He proved the existence of the limit of a projective systems of measurable spaces indexed by the set of natural numbers, where the spaces are Lusin measurable and the projection mappings are merely measurable. The main idea to prove it is to reduce the context, via measurable isomorphisms, to compact spaces such that the projections are continuous (hence admitting a projective limit); another proof of this result may be found in [DeMe 78], Chapter III, page 70. We emphasize that, in contrast with our approach, the metrizability of the underlying spaces plays a key role in the proofs of the previous mentioned versions, since one of the ingredients is Souslin-Lusin theorem on direct images of measurable, respectively Souslin sets. We also refer to [Rao 71] (Theorems 4.3, 4.5, and 4.7) for several characterizations for the existence of projective limits in terms of μ-pure fields, uniform σ-additivity, uniform integrable martingales, and Orlicz spaces.

The organization of the material of Chapter 5 is the following: in Section 5.1, after some preliminaries, we state the main results. In Section 5.2 we present the announced application needed in the construction of a branching process associated to a fragmentation one.

References

[AlKoRö 97a] S. Albeverio, Y. G. Kondratiev and M. Röckner, Ergodicity of L^2-semigroups and extremality of Gibbs states, *J. Funct. Anal.* 144 (1997) 394–423.

[AlKoRö 97b] S. Albeverio, Y. G. Kondratiev and M. Röckner, Ergodicity for the stochastic dynamics of quasi-invariant measures with applications to Gibbs states, *J. Funct. Anal.* 149 (1997) 415–469.

[BaHs 90] R.F. Bass and P. Hsu The semimartingale structure of reflecting Brownian motion", *Proc. Amer. Math. Soc.*, **108**, 1007–1010 (1990).

[Bert 06] J. Bertoin, *Random Fragmentation and Coagulation Processes*, Cambridge University Press, Ney York 2006.

[BeCî 14] L. Beznea and **I. Cîmpean**, On Bochner-Kolmogorov Theorem, *Séminaire de Probabilités XLVI* (Lecture Notes in Mathematics, Vol. 2123), Springer 2014, pp. 61–70.

[BeCî 16] L. Beznea and **I. Cîmpean**, Quasimartingales associated to Markov processes, *preprint*, 2016.

[BeCîRö 15] L. Beznea, **I. Cîmpean**, and M. Röckner, Irreducible recurrence, ergodicity, and extremality of invariant measures for resolvents, arXiv:1409.6492v2.

[BeCîRö 15a] L. Beznea, **I. Cîmpean**, and M. Röckner, A new approach to the existence of invariant measures for Markovian semigroups, arXiv:1508.06863v3.

[BeDeLu 15] L. Beznea, M. Deaconu, and O. Lupaşcu: Branching processes for the fragmentation equation. *Stochastic Processes and their Applications* **125** (2015), 1861–1885.

[BeDeLu 15a] L. Beznea, M. Deaconu, and O. Lupaşcu: Stochastic equation of fragmentation and branching processes related to avalanches. *J. of Statistical Physics* 2015 (to appear).

[Boch 55] S. Bochner, *Harmonic Analysis and the Theory of Probability*, Univ. California Press, Los-Angeles 1955.

[BoRöZh 00] V. Bogachev, M. Röckner, and T.S. Zhang, Existence and uniqueness of invariant measures: an approach via sectorial forms, *Appl. Math, Optim.* **41**, 87–109 (2000).

[ChFiWi 93] Z.Q. Chen, P.J. Fitzsimmons, R.J. Williams, Reflecting Brownian motions: quasimartingales and strong Caccioppoli sets, *Potential Analysis* **2**, 219–243 (1993).

[ChenFu 11] Z.Q. Chen and M. Fukushima, *Symmetric Markov Processes, Time Change, and Boundary Theory*, (Princeton University Press, 2011)

[ÇiJaPrSh 80] E. Çinlar, J. Jacod, P. Protter, and M.J. Sharpe, Semimartingale and Markov processes, *Z. Wahrscheinlichkietstheorie verw. Gebiete* **54**, 161–219 (1980).

[DaZa 96] G. Da Prato and J. Zabczyk, *Ergodicity for Infinite Dimensional Systems*, (Cambridge University Press, 1996).

[DeMe 78] C. Dellacherrie and P.A. Meyer, *Probabilities and Potential A*, Hermann, Paris 1978.

[DoFoGu 09] R. Douc, G. Fort, and A. Guillin, Subgeometric rates of convergence of f-ergodic strong Markov processes, Stoch. Proc. Appl. **119**, 897-923 (2009).

[Fu 99] M. Fukushima, On semi-martingale characterization of functionals of symmetric Markov processes *Electronic Journal of Probability*, **4**, 1–32 (1999).

[Fu 00] M. Fukushima, BV functions and distorted Ornstein Uhlenbeck processes over the abstract Wiener space *J. Funct. Anal.* **174**, 227–249 (2000)

[Fu 07] M. Fukushima, *Transience, Recurrence and Large Deviation of Markov Processes*, (Bielefeld IGK Seminar, 2007).

[FuOsTa 11] M. Fukushima, Y. Oshima, and M. Takeda, *Dirichlet forms and Symmetric Markov processes*, (Walter de Gruyter, Berlin/New York, 2011).

[Get 80] R. K. Getoor, Transience and recurrence of Markov processes, *Séminaire de probabilités (Strasbourg)* 14 (1980) 397–409.

[Hai 10] M. Hairer, Convergence of Markov Processes, Lecture Notes, University of Warwick, http://www.hairer.org/notes/Convergence.pdf, 2010.

[Hino 98] M. Hino, Existence of invariant measures for diffusion processes on a Wiener space, *Osaka J. Math.* **35**, 717–734 (1998).

[Hino 00] M. Hino, Exponential decay of positivity preserving semigroups on L^p, *Osaka J. Math.* **37**, 603–624 (2000).

[Kom 67] J. Komlós, A generalization of a problem of Steinhaus, *Acta Math. Acad. Sci. Hungar.* **18**, 217-229 (1967).

[KoPeSz 10] T. Komorowski, S. Peszat, and T. Szarek, On ergodicity of some Markov processes, *Ann. Probab.* **38**, 1401-1443 (2010).

[KoPaRö 12] Y. Kondratiev, T. Pasurek, and M. Röckner, Gibbs measures of continuous systems: An analytic approach, *Rev. Math. Phys.* **24** (2012), 1-54.

[LaSz 06] A. Lasota, and T. Szarek, Lower bound technique in the theory of a stochastic differential equation, *J. Differ. Equ.* **231**, 513–533 (2006).

[Liu 09] W. Liu, Harnack inequality and applications for stochastic evolution equations with monotone drifts, *J. Evol. Equ.* **9**, 747–770 (2009).

[MaUeWa 12] J. Masamune, T. Uemura and J. Wang, 'On the conservativeness and the recurrence of symmetric jump-diffusions', *J. Funct. Anal.* 263 (2012) 3984–4008.

[MeTw 93] S. P. Meyn and R. L. Tweedie, *Markov Chains and Stochastic Stability*, (Springer-Verlag, London, 1993).

[MeTw 93b] S.P. Meyn, and R.L. Tweedie, Stability of markovian processes II: continuous-time processes and sampled chains, *Adv. Appl. Prob.* **25**, 487–517 (1993).

[MeTw 93c] S.P. Meyn, and R.L. Tweedie, Stability of markovian processes III: Foster-Lyapunov criteria for continuous-time processes, *Adv. Appl. Prob.* **25**, 518–548 (1993).

[MeTw 93d] S.P. Meyn, and R.L. Tweedie, Generalized resolvents and Harris Recurrence of Markov processes, *American Mathematical Society, Providence, RI*, 227–250 (1993).

[Norr 97] J. R. Norris, *Markov Chains*, (Cambridge University Press, 1997).

[Oshi 92] Y. Oshima, 'On conservativeness and recurrence criteria for Markov processes', *Potential Anal.* 1 (1992) 115–131.

[Parth 67] K. R. Parthasarathy, *Probability Measures on Metric Spaces*, Academic Press, New York 1967.

[Pres 05] C. Preston, *Specifications and their Gibbs States*, Lecture Notes, Bielefeld University 2005.

[Rao 71] M. M. Rao, Projective limits of probability spaces, *J. Multivar. Anal*, **1** (1971), 28–57.

[Schi 04] R. L. Schilling, A note on invariant sets, *Probab. Math. Statist.* 24 (2004) 47–66.

[Sturm 94] K.T. Sturm, 'Analysis on local Dirichlet spaces. I. Recurrence, conservativeness and Lp-Liouville properties', *J. Reine Angew. Math.* 456 (1994) 173–196.

[Tw 01] R.L. Tweedie, Drift conditions and invariant measures for Markov chains, *Stoch. Proc. Appl.* **92**, 345–354 (2001).

[Wa 13] F.-Y. Wang, *Harnack Inequalities for Stochastic Partial Differential Equations*, Springer, New York, 2013.