PROJET DE RECHERCHE

FLORIN AVRAM, DANIEL MATEI

Titre: *Methodes algébriques en probabilités et statistique.*

Participants: Florin Avram (Université de Pau et des Pays de l'Adour) et Daniel Matei (Institut de Mathématiques "Simion Stoilow" Bucarest).

Déscription du projet:

Algebraic methods have often wrought miracles in probability and statistics – see for example Diaconis [8]. Towards the end of the last century, some exciting cross-fertilization between the two fields has arisen notably by collaborations between Diaconis, Eisenbud and Sturmfels [9, 10]. A new field was born, as certified by the recent book [19] of Pistone, Riccomagno, and Wynn (2001) – see also [20] for a survey. Stimulated by the advent of modern algebraic symbolic computing, a whole "algebra educated" community attacked old problems arising in experimental design, counting contingency tables, parameter estimation, hypothesis testing, simulation, biology models and asymptotic analysis — see for example [18]. As the literature in French on the related statistical aspects is rather scarce (see though Bertrand [2]), we decided to make this presentation in English.

We propose to consider several proba-stat applications, where it is hoped that a joint investigation by algebraists and probabilists will turn out fruitful.

1. Our first proposal is to explore the utility of *Lie-algebraic methods* (e.g. Baker-Campbell-Hausdorff, Magnus, and Wei-Norman formulas) in providing *asymptotic expansions* of interest in probability and statistics. It seems natural that Lie-algebraic methods should yield useful expansions for Markovian semigroups, for example in the study of *polynomial processes* recently introduced by Cuchiero, Keller-Ressel and Teichmann [6]. However, the algebraic approach has not been popular in the study of stochastic processes, at least until the recent numerical integration methods from [17, 14, 16, 12], and the appearance of the "rough paths" theory – see for example [15] by Lyons.

2, 3. Our second interest is the application of *hyperplane arrangements* theory to *asymptotics*, related to recent works by Baryshnikov and Pemantle [1], Brion and Vergne [3, 4], and Szenes and Vergne [22], as well as to *maximum likelihood estimation*, related to the recent work of Catanese, Hoşten, Khetan, and Sturmfels [5] – see also [13, 11, 21].

Activités et objectif du projet:

• F. Avram will give a series of lectures on *Algebraic methods in mathematical finance and probability* at IMAR; D. Matei will give a series of lectures on *Algebraic statistics* at UPPA.

• We expect our collaboration will result in writing at least one paper on the subject.

Visites envisagées:

• F. Avram à Bucarest - 2 semaines en août 2010

• D. Matei à Pau - 2 semaines en avril 2011

Financement demandé au L.E.A.: Total 2200 euro.

Frais de voyage (400 euro), frais de logement (hotel, 60 euro par jour, total 840 euro) et perdiem (15 euro par jour, total 210 euro) pour 2 semaines de séjour en Roumanie (Bucarest), pour un total de 1450 euro.

Frais de logement (330 euro, résidence universitaire) et perdiem (30 euro par jour, total 420 euro) pour 2 semaines de séjour en France (Pau), pour un total de 750 euro. Les frais de voyage de D. Matei seront supportés par IMAR sur un contrat de recherche.

References

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