

Proposal in the framework of the French-Romanian laboratory

# Existence of travelling waves for nonlocal reaction-diffusion equations

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## 1 Scientific context

The project is devoted to the integro-differential systems of equations of the type

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + F(u, J(u)), \quad (1.1)$$

where  $J(u)$  is the convolution product

$$J(u) = \int_{-\infty}^{\infty} \phi(x-y)u(y, t)dy.$$

Such problems arise in numerous applications including population dynamics with nonlocal consumption of resources or nonlocal stimulation of reproduction [6]. Travelling wave solution of this system is a solution of the form  $u(x, t) = w(x - ct)$ . It satisfies the system

$$w'' + cw' + F(w, J(w)) = 0 \quad (1.2)$$

with some limits at infinity  $w(\pm\infty) = w_{\pm}$ .

If we replace the kernel  $\phi$  of the integral by the  $\delta$ -function, then we obtain the usual reaction-diffusion system of equations. Existence of travelling waves for the scalar reaction-diffusion equation is studied in detail. There are also some classes of systems for which the existence can be proved. In particular, for monotone systems characterized by the applicability of the maximum principle.

In the case of the scalar integro-differential equation, the wave existence is proved by the method of lower and upper functions [5]. However this method is not applicable for systems of equations. In our previous works we proved wave existence in the case of small support of the kernel  $\phi$  where some perturbation methods can be used [2], [3]. Some related questions are studied in the works [4], [1]. In the framework of this project we propose to develop a new approach based on the topological degree and Leray-Schauder method.

## 2 Expected results

1. We will find the essential spectrum of linear integro-differential operators and will determine the conditions when the operators satisfy the Fredholm property. It will be done on the basis of the theory of limiting operators developed for elliptic problems in unbounded domains and adapted for the problems under consideration. Solvability conditions for linear equations in terms of orthogonality to solutions of the homogeneous adjoint equation will be proved. Behavior of solutions at infinity will be studied. The index of the operator will be determined,
2. Properness of the nonlinear operators in weighted Hölder spaces will be proved and the construction of the topological degree for Fredholm and proper operator with zero index will be applied,
3. A priori estimates of solutions will be obtained and the Leray-Schauder method will be used in order to prove existence of travelling waves.

## 3 Existing collaboration

The participants of this project have 6 joint publications, 3 of them are directly related to the topic of the proposal. They were coordinators of two French-Romanian projects PAI. The last one in 2007-2008.

## 4 Required financial support

We propose this work for two years, 2010 and 2011. It will require mutual visits of the two participants. We intend to have one visit of the Romanian participant to France and one visit of the French participant to Romania each year. Each visit is for two weeks. We suggest to support the Romanian participant in the framework of the French-Romanian laboratory. The visits of the French participant to Romania will be payed from other grants.

The estimate of the cost: 500 euros for travel expenses, 1000 euros for living expenses in France. Total for 2010: 1500 euros. Total for 2010-2011: 3000 euros.

## References

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