A qualitative description of travelling shallow water waves with constant vorticity

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Abstract. This document contains a brief description of our collaborative research project which is submitted to the French–Romanian GDRI.

Key words and phrases: shallow water waves; waves with constant vorticity; phase plane analysis; qualitative analysis of ODEs

Research context

During our previous project financed by LEA Math-Mode in 2015/2016 (the last year of LEA functioning) we performed a unified procedure to derive and to study the travelling wave solutions to several two-component systems modelling shallow water waves: the Green-Naghdi system, the integrable two-component Camassa-Holm equations and a new two-component system [2] of Green-Naghdi type. In particular, we were interested in solitary and cnoidal-type solutions, as two most important classes of travelling waves that one encounters in applications. We provided a complete phase-plane analysis of all possible travelling wave solutions which may arise in the models above. In particular, we showed the existence of new type of solutions. Our work was published in Journal of Differential Equations in 2016 [1].

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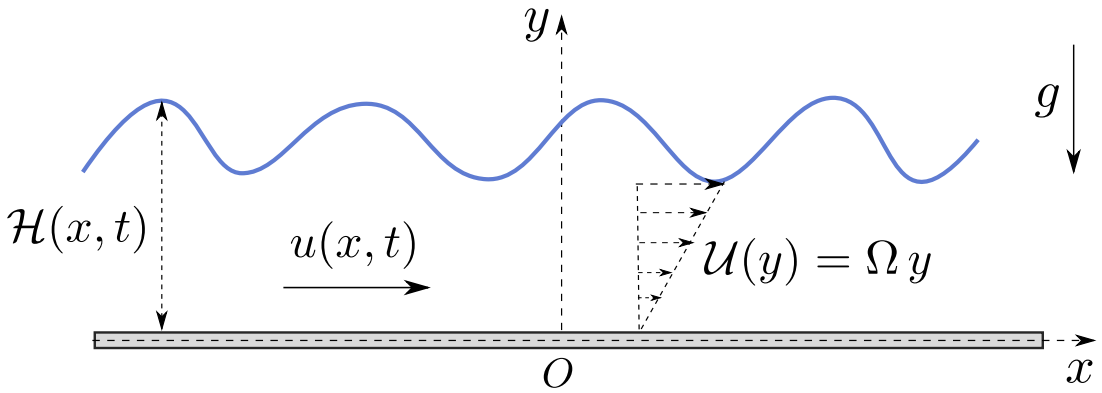

Figure 1. *Sketch of the fluid domain along with the internal flow structure.*

In the present proposal we would like to extend our analysis to a more general situation of water waves over a background current. Mathematically it corresponds to model water waves with constant vorticity. The sketch of the fluid domain is shown in Figure 1. Namely, we would like to consider the following three models for such waves: two-component Camassa-Holm equations with constant vorticity, Zakharov-Itō system and Kaup-Boussinesq system. The goal is to produce a similar classification to that published in [1] for waves without current. We plan to use the methods of the phase plane analysis for ODEs describing the travelling wave solutions. We also intend to investigate further the nonlinear phenomena related to the dynamics of the travelling waves with constant vorticity, for example, by numerical methods as in [5].

Project budget

We would like to ask the GDRI to cover the fees related to mutual visits of participants to France and to Romania respectively:

D. Ionescu-Kruse: One round trip from Bucharest (Romania) to Chambéry (France) and a continuous stay at LAMA UMR 5127 during one week in 2018

D. Dutykh: Two round trips from Chambéry (France) to Bucharest (Romania) and a continuous stay at IRMAR during two weeks: one week in 2017 and another one in 2018.

The travel fee is estimated to be around 350 € and the *per diem* is 90 €.

5 Recent articles of the project participants

[1] D. Dutykh, D. Ionescu-Kruse, Travelling wave solutions for some two-component shallow water models, *J. Differential Equations* **261** (2016), 1099–1114. 1, 2

[2] D. Ionescu-Kruse, A new two-component system modelling shallow-water waves, *Quart. Appl. Math.* **73** (2015), 331–346. 1

[3] D. Ionescu-Kruse, Variational derivation of a geophysical Camassa-Holm type shallow water equation, *Nonlinear Anal.* **156** (2017), 286–294.

[4] D. Mitsotakis, D. Dutykh, A. Assylbekuly, D. Zhakebayev, On weakly singular and fully nonlinear travelling shallow capillary-gravity waves in the critical regime, *Phys. Lett. A* **381** (2017), 1719–1726.

[5] D. Mitsotakis, D. Dutykh, J. Carter, On the nonlinear dynamics of the traveling-wave solutions of the Serre system, *Wave Motion* **70** (2017), 166–182. 2

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