Research projet - team 4

Multiplier properties on Hilbert spaces of analytic functions

Participants

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Scientific project

We intend to consider Hilbert spaces of analytic functions on the unit disc, among which:

- Model spaces, that is spaces of the form $K_\Theta = H^2 \ominus \Theta H^2$, where Θ is an inner function.
- de Branges–Rovnyak spaces, which are a generalization of model spaces for the case when Θ is no more inner.
- Spaces with the complete Pick property—these are spaces for which a Nevanlinna–Pick type interpolation result is true; they are characterized by a special property of their reproducing kernel.

All these spaces (as well as their generalizations) play a central role in operator theory. We are interested by a special class of operators related to these spaces, namely multiplication operators, either on the same space or mapping one space into another. This study has already been initiated in [4, 6, 5],

The first direction of investigation of the project consists of obtaining, for various spaces, conditions for multipliers to be bounded, compact, or in certain Schatten–von Neumann classes. This question is related with delicate analytic and topological questions related to the spaces of holomorphic functions, as one can see in [2].

In connection with multipliers, a recent direction of investigation has been the consideration of Smirnov classes associated to spaces of analytic functions. These are formed by quotients of multipliers of the given spaces. Characterization of these classes have been obtained in

particular cases in [1, 7]. We are interested in specific properties of such a class for other Hilbert spaces of analytic functions having more structure, such as for instance de Branges-Rovnyak spaces.

References

[1] A. Aleman, M. Hartz, J. McCarthy & St. Richter, *The Smirnov class for spaces with the complete Pick property*, preprint, https://arxiv.org/abs/1701.07476

[2] A. Baranov, *Compact embeddings of model subspaces of the Hardy space*, preprint, https://arxiv.org/abs/0712.0684v1.

[3] C. Chu, Which de Branges-Rovnyak spaces have complete Nevanlinna-Pick property?, preprint, http://arxiv.org/abs/1910.04830v1.

[4] E. Fricain, A. Hartmann, W.T. Ross, *Multipliers between model spaces*. Studia Math. **240** (2018), no. 2, 177–191.

[5] E. Fricain, R. Rupam, *A note on multipliers between model spaces*, New York J. Math. 24 (2018), 147–156.

[6] E. Fricain, A. Hartmann, W.T. Ross, *Multipliers between range spaces of co-analytic Toeplitz operators*, Acta Sci. Math. (Szeged) 85 (2019), no. 1-2, 215–230.

[7] M. Jury & R. Martin, *The Smirnov classes for the Fock space and complete Pick spaces*, preprint, https://arxiv.org/abs/1806.05270

[8] S. Shimorin, *Complete Nevanlinna-Pick property of Dirichlet-type spaces*, J. Funct. Anal. 191 (2002), no. 2, 276–296.

Visits

The project will cover the two years 2020–2021. In each of the years there will be a one week visit of Dan Timotin in France (at the University of Lille or Bordeaux) and a one week visit of Emmanuel Fricain and Andreas Hartmann at the Simion Stoilow Institute of Mathematics in Bucharest.