Gerald Trutnau: On the generator of a right-continuous Markov process.

Abstract: (Joint work with Michael Röckner, Lucian Beznea) Given a right-continuous Markov process $(X_t)_{t\geq 0}$ on a second countable metrizable space E with transition semigroup $(p_t)_{t\geq 0}$, we prove that there exists a σ-finite Borel measure μ with full support on E, and a closed and densely defined linear operator $(\mathcal{L}_p, D(\mathcal{L}_p))$ generating $(p_t)_{t\geq 0}$ on $L^p(E;\mu)$. In particular, we solve the corresponding Cauchy problem in $L^p(E;\mu)$ for any initial condition $u \in D(\mathcal{L}_p)$. Furthermore, for any real $\beta > 0$ we show that there exists a generalized Dirichlet form which is associated to $(e^{-\beta t}p_t)_{t\geq 0}$. If the β-subprocess of $(X_t)_{t\geq 0}$ corresponding to $(e^{-\beta t}p_t)_{t\geq 0}$, $\beta > 0$, is μ-special standard then all results from generalized Dirichlet form theory become available, and Fukushima's decomposition holds for $u \in D(\mathcal{L}_2)$. The question under which additional assumptions the β-subprocess becomes μ-special standard shall hence also be studied. If $(X_t)_{t\geq 0}$ is transient, then β can be chosen to be zero.