Zoltan Balogh: Gromov's dimension comparison and fractal geometry on Carnot groups.

Abstract: We solve Gromov's dimension comparison problem for Hausdorff and box counting dimension on Carnot groups equipped with a Carnot-Carathéodory metric and an adapted Euclidean metric. The proofs use sharp covering theorems relating optimal mutual coverings of Euclidean and Carnot-Carathéodory balls, and elements of sub-Riemannian fractal geometry associated to horizontal self-similar iterated function systems on Carnot groups. Inspired by Falconer's work on almost sure dimensions of Euclidean self-affine fractals we show that Carnot-Carathéodory self-similar fractals are almost surely horizontal. As a consequence we obtain explicit dimension formulae for invariant sets of Euclidean iterated function systems of polynomial type.